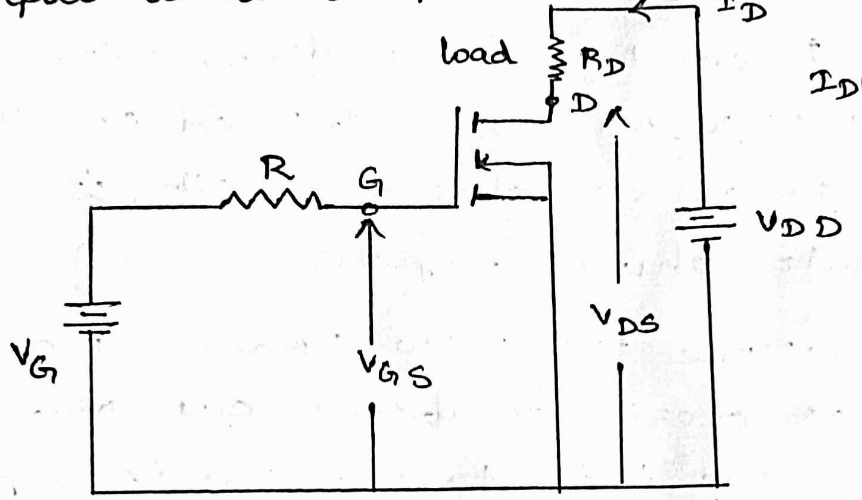
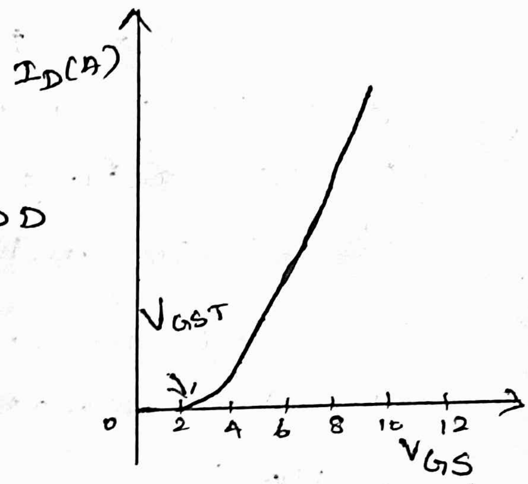
POWER MOSFET characteristics :

[The basic circuit diagram for n-channel PMOSFET is shown in figure, where voltage and currents are as indicated. The source terminal S is taken as common terminal, between input and output of a MOSFET].



N channel P MOSFET circuit diagram



Transfer characteristics

Transfer characteristics :

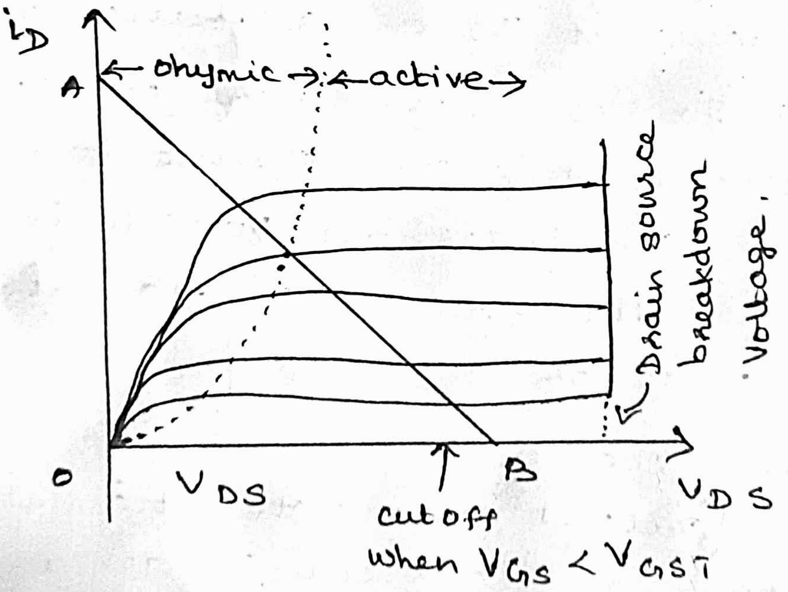
[This characteristics shows the variation of drain current I_D as a function of gate source voltage V_{GS}. V_{GST} is the minimum positive voltage between gate and source to induce n-channel. For threshold voltage below V_{GST}, device is in the off-state. Magnitude of V_{GST} is of the order of 2 to 3 V.]

Output characteristics :

[Output characteristics indicate the variation of drain current I_D as a function of drain source voltage V_{DS}. For low values of V_{DS}, the graph between I_D - V_{DS} is almost linear. This indicates a constant value of on-resistance $R_{DS} = V_{DS} / I_D$. For given V_{GS}, if V_{DS} is increased, output characteristic is relatively flat, that drain current is nearly constant. A load line intersects the output characteristics at A and B. Here A indicates fully ON condition, B indicates fully off state. PMOSFET operates as a switch either at A or B].

When Power MOSFET is driven with large gate source voltage, MOSFET is turned ON. The MOSFET acting as a closed switch, is said to be driven into ohmic region.

When device turns ON, PMOSFET traverses $i_D - V_{DS}$ characteristics from cut-off to active region and then to the ohmic region. When PMOSFET turns off, it takes backward journey from ohmic region to cut-off state.



output characteristics of PMOSFET.

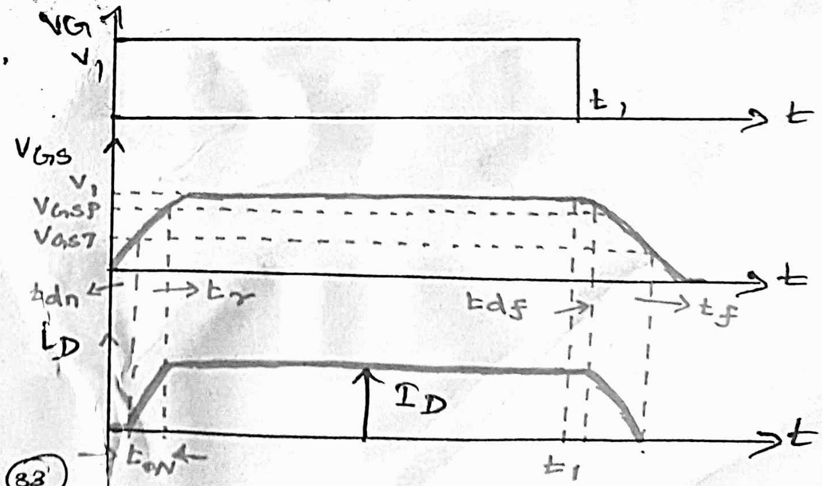
Switching characteristics :

[The switching characteristics of a power MOSFET are influenced to a large extent by the internal capacitance of the device and the internal impedance of the gate drive circuit].

[At turn-on, there is an initial delay t_{dn} during which input capacitance charges to gate threshold voltage V_{GST}. Here t_{dn} is called turn-on delay time.]

[There is further delay, t_r called rise time, during which gate voltage rises to V_{GSP}, a voltage sufficient to drive the MOSFET into ON state. During t_r, drain current rises from zero, to full-on current I_D. The total turn on time is, $t_{on} = t_{dn} + t_r$].

[Turn off process is initiated soon, after removal of gate voltage at time t_1. The turn-off delay time, t_{df}, is the time during which input capacitance discharges from overdrive gate voltage V_1 to V_{GSP}]. Fall time t_f is the time during which input capacitance discharges from V_{GSP} to threshold voltage.



Switching waveforms for PMOSFET.

Switching Mode Regulators

DC Converters can be used as switching mode regulators to convert a dc voltage, normally unregulated to a regulated dc output voltage.

There are four basic topologies of switching Regulators :

1. Buck Regulators
2. Boost Regulators
3. Buck - Boost Regulators
4. cuk Regulators.

Buck Regulator :- [step down converter]

The average output voltage V_a is less than the input voltage V_s, hence the name buck, a very Popular regulator.

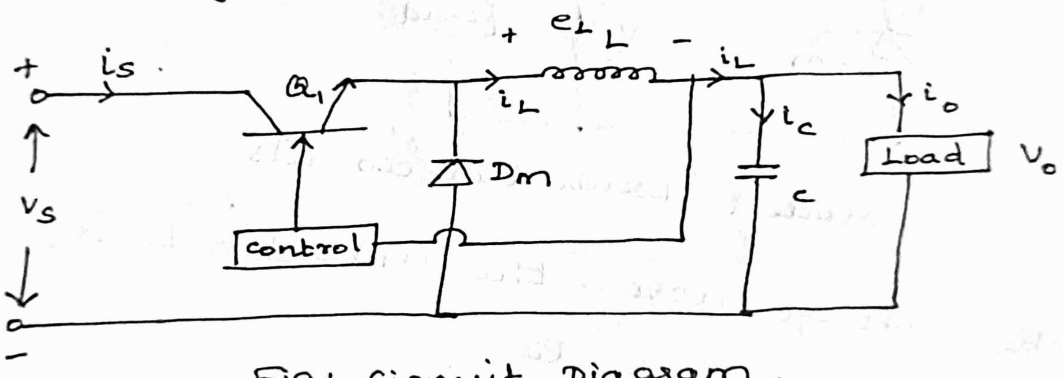
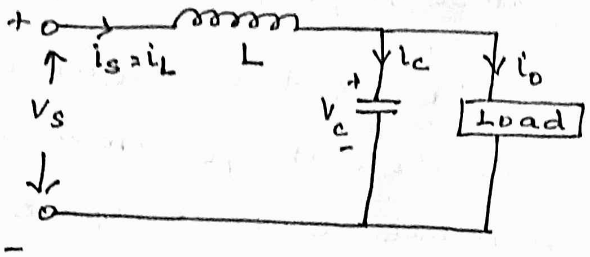


Fig: Circuit Diagram.

The circuit diagram of a buck regulator using a Power BJT is shown in fig. circuit operation can be divided into two modes.

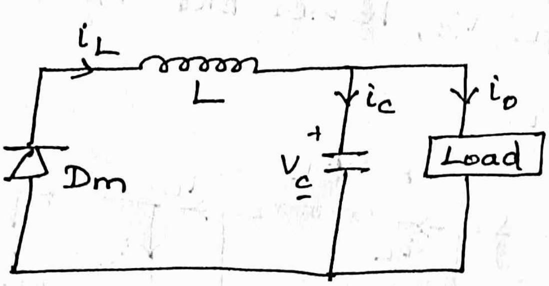
Mode I : Mode I begins when transistor Q_1 is switched on at $t = 0$.



Mode 1 - Equivalent circuit

The input current which raises, flows through filter inductor L, filter capacitor C, load Resistor R.

Mode 2 : Mode 2 begins when transistor Q_1 is switched off at $t = t_1$. The freewheeling Diode Dm conducts due to energy stored in the inductor. The inductor current continues to flow through L, C, load and diode Dm.



Mode 2 - Equivalent circuits.

The voltage across the inductor L is,

$$e_L = L \cdot \frac{di}{dt}$$

Assuming inductor current rises linearly from I_1 to I_2, in time t_1.

$$V_s - V_a = L \cdot \frac{(I_2 - I_1)}{t_1} \quad \text{——— (1)}$$

$$\boxed{\Delta I = I_2 - I_1}$$

$$= L \cdot \frac{\Delta I}{t_1}$$

$\Delta I \rightarrow$ Peak to peak ripple current of inductor

$$\boxed{t_1 = \frac{\Delta I \cdot L}{V_s - V_a}} \quad \text{——— (2)}$$

Inductor current falls linearly from I_2 to I_1 in time t_2,

from eqn ①,

$$-V_a = -L \cdot \frac{\Delta I}{t_2} \qquad [\because Q_1 \to \text{off}, \ V_s = 0].$$

$$\boxed{t_2 = \frac{\Delta I \cdot L}{V_a}} \quad \text{——— (3)}$$

Equating the value of ΔI in eqns (2) & (3).

$$② \Rightarrow \quad \Delta I = \frac{(V_s - V_a)\, t_1}{L}$$

$$③ \Rightarrow \quad \Delta I = \frac{V_a t_2}{L}.$$

$$\Delta I = \frac{(V_s - V_a)\, t_1}{\cancel{L}} = \frac{V_a t_2}{\cancel{L}}.$$

$$\Delta I = (V_s - V_a)\, t_1 = V_a t_2.$$

sub $t_1 = KT$, $t_2 = (1-K)T$, the average output voltage is,

$$(V_s - V_a)\, KT = V_a (1-K)\, T.$$

$$V_s k - V_a k = V_a - V_a k.$$

$$\boxed{t_1 = KT \quad K = \frac{t_1}{T}}$$

$$V_s k = V_a.$$

$$\boxed{V_a = V_s \frac{t_1}{T} = k V_s} \quad \text{——— (4).}$$

Assuming a lossless circuit,

$$V_s I_s = V_a I_a$$

Sub $V_a = kV_s$.

$$\cancel{V_s} I_s = k \cancel{V_s} I_a .$$

$$\boxed{I_s = k I_a .} \quad \text{———————} (5).$$

Switching period T can be expressed as,

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_s - V_a} + \frac{\Delta I L}{V_a} \qquad \text{(from (2) \& (3))}$$

$$= \frac{\Delta I . L . V_a + \Delta I . L (V_s - V_a)}{V_a (V_s - V_a)}$$

$$= \cancel{\Delta I . L . \Sigma V_a}$$

$$= \frac{\Delta I . \cancel{L V_a} + \Delta I . L V_s - \Delta I . \cancel{L V_a} .}{V_a (V_s - V_a)}$$

$$\boxed{T = \frac{\Delta I . L . V_s}{V_a (V_s - V_a)}} \quad \text{———————} (6).$$

Peak to peak ripple current as,

$$\Delta I = \frac{V_a (V_s - V_a) T}{L . V_s .} \qquad T = 1/f .$$

$$\boxed{\Delta I = \frac{V_a (V_s - V_a)}{f . L . V_s}} \quad \text{———————} (7).$$

using kirchoff's current law,

$$i_L = i_c + i_o .$$

$$\Delta i_L = \Delta i_c + \Delta i_o . \qquad \Delta i_o \rightarrow \text{is very small, and negligible.}$$

$$\Delta i_L = \Delta i_c .$$

The average capacitor current flows into for,

$$\frac{t_1}{2} + \frac{t_2}{2} = T/2 \text{ is,}$$

$$I_c = \frac{\Delta I}{4}.$$

capacitor voltage is expressed as,

$$V_c = \frac{1}{c}\int i_c . dt + V_c\,(t=0).$$

Peak to peak ripple voltage of the capacitor is,

$$\Delta V_c = V_c - V_c\,(t=0)$$

$$= \frac{1}{c}\int_0^{T/2} \frac{\Delta I}{4}\, dt.$$

$$= \frac{1}{c}\frac{\Delta I}{4}\,[t]_0^{T/2}$$

$$\Delta V_c = \frac{1}{c}\frac{\Delta I}{4} \times T/2 = \frac{\Delta I . T}{8C} = \frac{\Delta I}{8fC}$$

$$\boxed{\Delta V_c = \frac{\Delta I}{8f\,C}} \quad \text{——— (8).}$$

Sub the value of ΔI from (7) in eqn (8).

$$\boxed{\Delta V_c = \frac{V_a\,(V_s - V_a)}{8 f^2 LC\, V_s}} \quad \text{——— (9).}$$

critical value of inductor $L_c = \frac{(1-k)R}{2f}$.

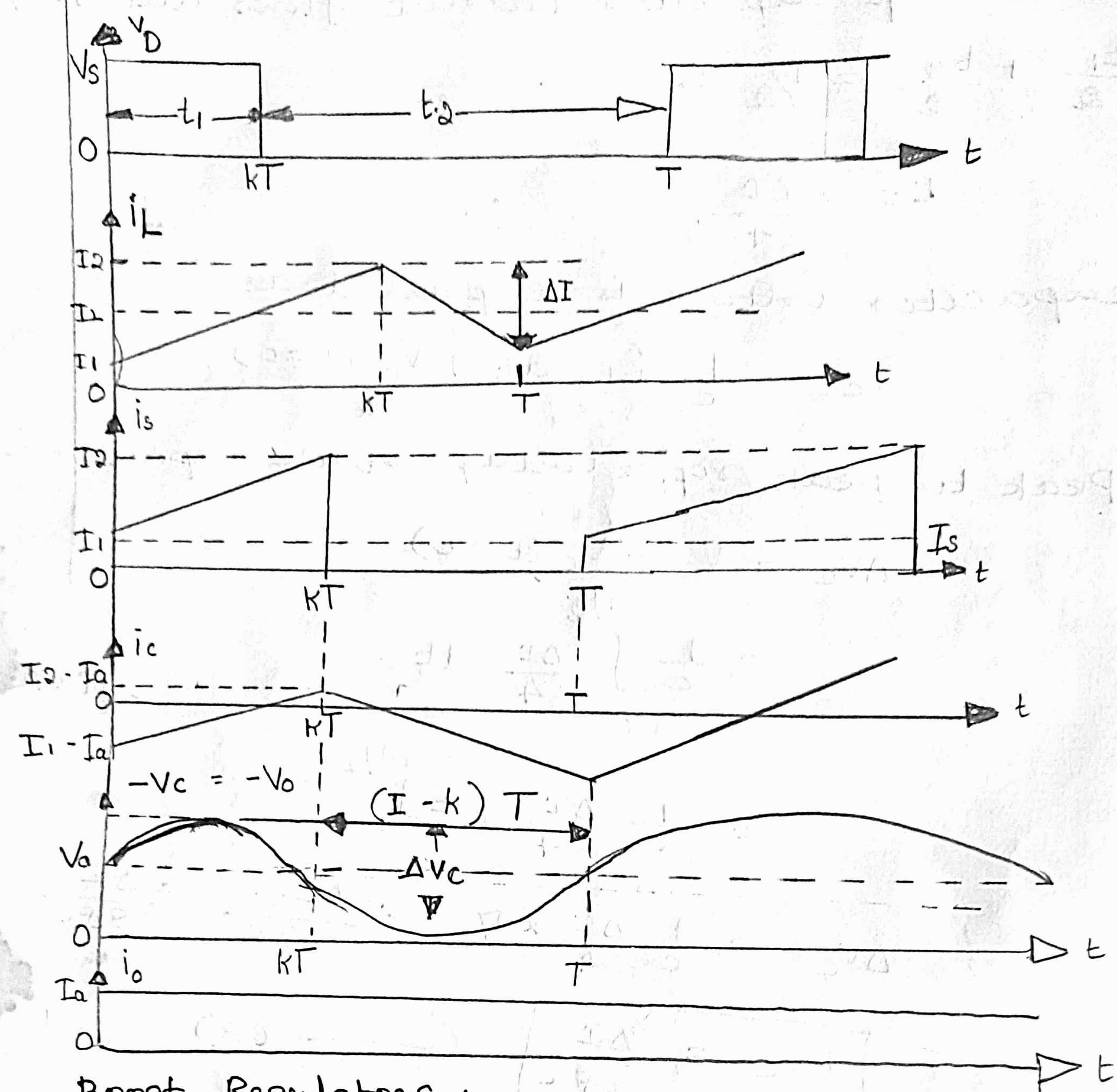
critical value of capacitor $C_c = \frac{1-k}{16Lf^2}$.

Advantages:

① It requires only one Transistor, simple, high efficiency greater than 90%, di/dt of load current limited by L.

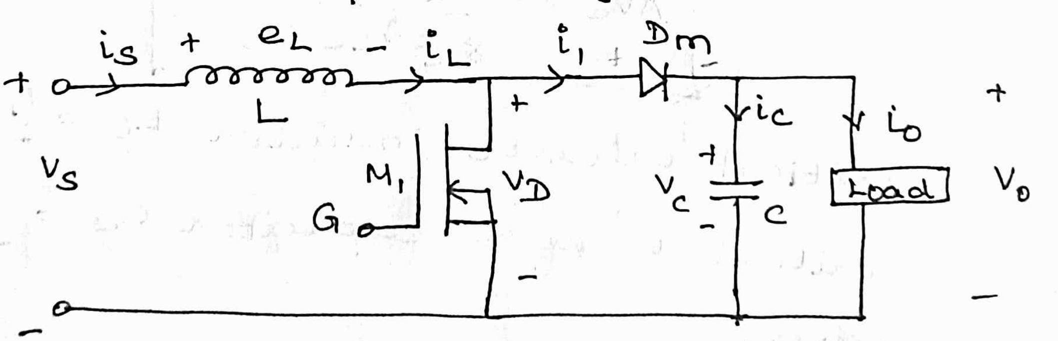
Disadvantages:

① Input current is discontinuous,
② smoothing filter required.
③ It provide one polarity of output voltage.
④ It requires a protection circuit.



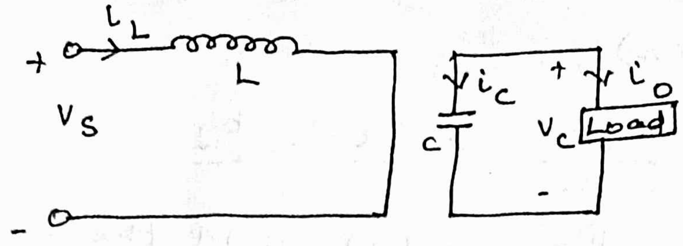
Boost Regulators :-

In a boost regulator, the output voltage is greater than the input voltage hence the name boost.



Circuit Diagram.

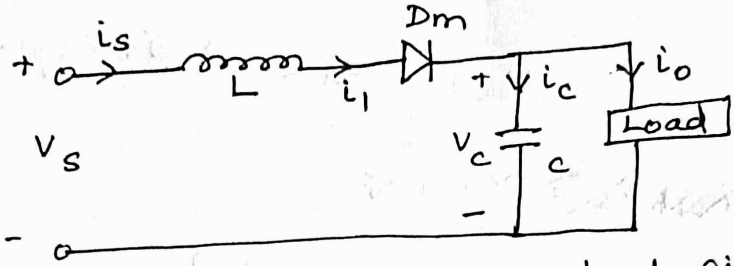
A boost regulator using a power MOSFET is shown in figure.



Mode-1 - Equivalent circuit

Mode1 :

Mode 1 begins when transistor M_1 is switched on at $t=0$. The input current which rises, flows through inductor L & transistor Q_1.



Mode 2 - Equivalent circuit.

Mode 2 begins when transistor M_1 is switched off at $t=t_1$. The current flow through L, C, load and diode Dm. The inductor current falls. The energy stored in inductor L is transferred to the load.

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1.

$$V_s = L\frac{(I_2-I_1)}{t_1} = L\cdot\frac{\Delta I}{t_1} \quad \text{——— (1).}$$

$$\boxed{t_1 = \frac{L\cdot\Delta I}{V_s}} \quad \text{——— (2).}$$

$\Delta I \rightarrow$ peak to peak ripple current of inductor L.

The inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_a - \cancel{V_a}\ V_s = -L\cdot\frac{\Delta I}{t_2} \quad \text{——— (3).}$$

$V_s - V_a$

$$\boxed{t_2 = \frac{+L\cdot\Delta I}{V_a - V_s}} \quad \text{——— (4).}$$

from (2) & (4),

$$\Delta I = \frac{V_s t_1}{L} = \frac{(V_a - V_s) t_2}{L}$$

sub $t_1 = kT$, $t_2 = (1-k)T$ yield the average output voltage,

$$\frac{V_s kT}{L} = \frac{(V_a - V_s)(1-k)T}{L}.$$

$$\frac{V_s kT}{\cancel{L}} = \frac{V_a T - V_a kT - V_s T + V_s \cdot kT}{\cancel{L}}$$

$$V_s \cancel{k}T = V_a T - V_a kT - V_s T + V_s \cancel{k}T.$$

$$V_s T = V_a T [1-k].$$

$$\boxed{V_a = \frac{V_s \cancel{k}}{\cancel{k}(1-k)}} \quad \text{——— (5).}$$

sub $k = \frac{t_1}{T} = t_1 f$ in eqn (5).

$$\boxed{V_a = \frac{V_s}{\cancel{k}(1 - t_1 f)}}.$$

Assuming a lossless circuit, $V_s I_s = V_a I_a$

$$\cancel{V_s} I_s = \frac{\cancel{V_s} \cdot I_a}{\cancel{k}(1-k)}$$

$$\boxed{I_s = \frac{I_a}{\cancel{k}(1-k)}} \quad \text{——— (6).}$$

The switching period T can be found from,

$$T = \frac{1}{f} = t_1 + t_2.$$

$$= \frac{\Delta I L}{V_s} + \frac{\Delta I L}{V_a - V_s}.$$

$$\boxed{T = \frac{\Delta I_L V_a}{V_s (V_a - V_s)}} \text{ —— (7).} \quad = \frac{\Delta I L (V_a - V_s) + \Delta I L \cdot V_s}{V_s (V_a - V_s)}.$$

$$\leftarrow T = \frac{\Delta I_L V_a - \Delta I \cancel{L} V_s + \Delta I \cancel{L} V_s}{V_s (V_a - V_s)}.$$

Peak to peak ripple current :

$$\Delta I = \frac{V_s (V_a - V_s) T}{L . V_a}$$

$$\boxed{\Delta I = \frac{V_s (V_a - V_s)}{f . L . V_a}} \quad \text{---(7)} \quad \therefore T = 1/f .$$

when the transistor is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time t_1 is $I_c = I_a$.

Peak to peak ripple voltage of the capacitor is,

$$\Delta V_c = \frac{1}{c} \int_0^{t_1} I_c . dt$$

$$= \frac{1}{c} \int_0^{t_1} I_a . dt = \frac{I_a t_1}{c} .$$

$$\text{sub } t_1 = \frac{V_a - V_s}{V_a f} .$$

$$\Delta V_c = \frac{I_a (V_a - V_s)}{c . V_a . f}$$

$$\boxed{\Delta V_c = \frac{I_a K}{f c}} . \quad \text{---(8)} .$$

condition for ~~maximum~~ continuous inductor current and capacitor voltage :

$$L_c = \frac{k (1-k) R}{2 f} .$$

$$C_c = \frac{k}{2 f R} .$$

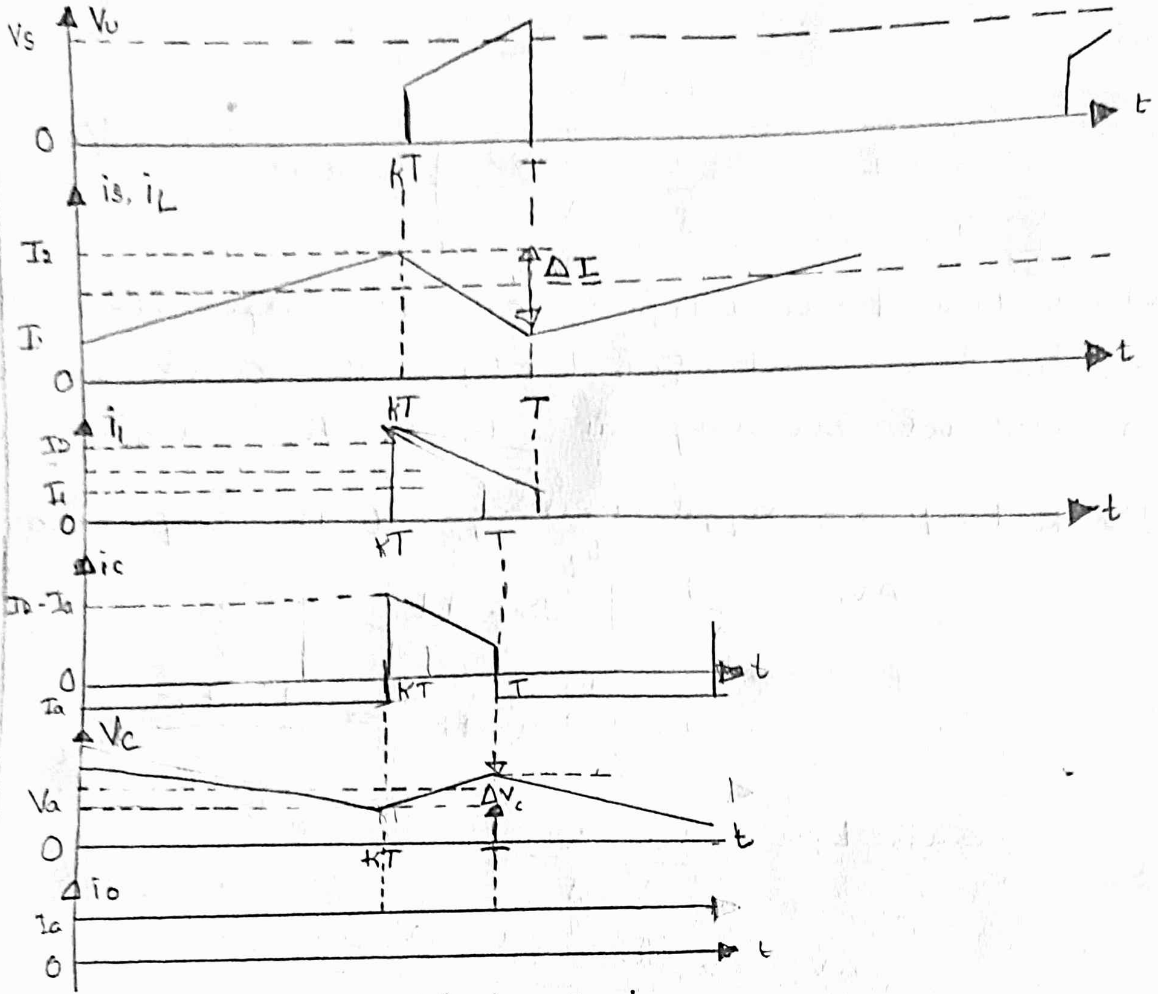
Advantages :

① step up the output voltage without a Transformer.
② High Efficiency . Input current continuous .

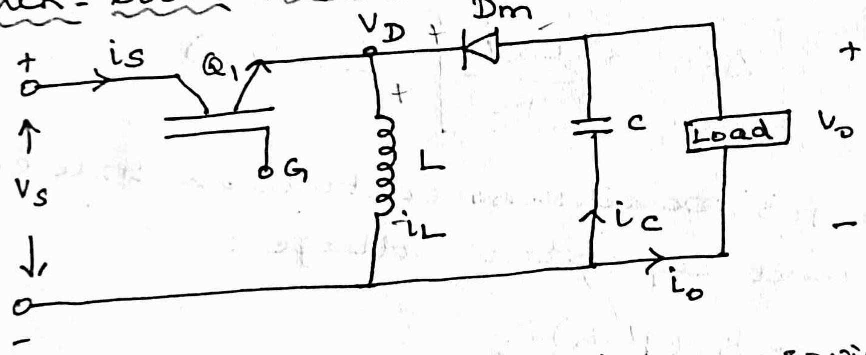
Disadvantages : -

① High peak current has to flow through the transistor
② Difficult to stabilize the regulator .
③ larger filter capacitor and inductor are required.

~~Buck - Boost Regulators~~:



Buck - Boost Regulators :

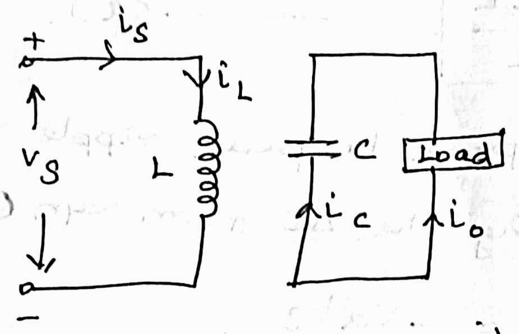


A Buck Boost regulator provides an output voltage that may be less than or greater than the input voltage - hence the name buck - boost. The output voltage polarity is opposite to that of the input voltage.

This regulator is also known as an inverting regulator.

Mode I :

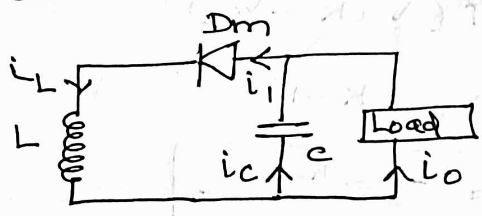
During mode 1, transistor Q_1 is turned on and diode Dm is reverse biased. The input current which rises, flows through inductor L, transistor Q_1.



Equivalent circuit

Mode II :-

During mode 2, transistor Q_1 is switched off and the current which was flowing through inductor and the current which was flowing through inductor L, C, Dm and the load. The energy stored in L would be transferred to the load and inductor current would fall.



Equivalent circuit.

Assuming the inductor current rises linearly from I_1 to I_2 in time t_1.

$$V_S = L\frac{(I_2 - I_1)}{t_1} = L\cdot\frac{\Delta I}{t_1}.$$

$$\boxed{t_1 = \frac{L\cdot \Delta I}{V_S}} \quad \text{———— (1)}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2.

$$V_a = -L \cdot \frac{\Delta I}{t_2}$$

$$\boxed{t_2 = \frac{-\Delta I \cdot L}{V_a}} \quad \text{------ (2)}$$

$\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor L. Equating the ΔI from eqn (1) & (2).

$$\Delta I = \frac{V_s t_1}{\cancel{L}} = \frac{-V_a t_2}{\cancel{L}}$$

sub $t_1 = kT$, $t_2 = (1-k)T$, the average output voltage is

$$V_s(kT) = -V_a(1-k)T$$

$$= -V_a(T-kT)$$

$$V_s kT = -V_a T + V_a kT$$

$$V_s k\cancel{T} = \cancel{T}(V_a k - V_a)$$

$$V_s k = V_a(k-1)$$

$$\boxed{V_a = \frac{-V_s k}{(1-k)}} \quad \text{------ (3)}$$

Assuming a lossless circuit, $V_s I_s = -V_a I_a$

$$\cancel{V_s} I_s = \frac{\cancel{V_s} k \cdot I_a}{(1-k)}$$

$$\boxed{I_s = \frac{I_a \cdot k}{1-k}} \quad \text{------ (4)}$$

The switching period T can be found from,

$$T = \frac{1}{f} = t_1 + t_2$$

$$= \frac{\Delta I L}{V_s} + \left(-\frac{\Delta I L}{V_a}\right)$$

$$= \frac{\Delta I L V_a - \Delta I L V_s}{V_s V_a}$$

$$T = \frac{\Delta I L [V_a - V_s]}{V_s V_a}$$

$$\Delta I = \frac{T \cdot V_s V_a}{L \cdot (V_a - V_s)}$$

$$\boxed{\Delta I = \frac{V_s \cdot V_a}{f \cdot L \cdot (V_a - V_s)}} \quad \text{—— (5)}.$$

when transistor Q_1 is on the filter capacitor supplies the load current for $t = t_1$. The average discharging current of the capacitor is $I_c = I_a$,

Peak to peak ripple voltage of the capacitor is,

$$\Delta V_c = \frac{1}{c}\int_0^{t_1} I_c \cdot dt = \frac{1}{c}\int_0^{t_1} I_a \cdot dt$$

$$\boxed{\Delta V_c = \frac{I_a t_1}{c}} \quad \text{—— (6)}$$

$$\Delta V_c = \frac{I_a V_a}{C f (V_a - V_s)} \quad \text{—— (7)}$$

Condition for continuous inductor current and capacitor voltage:

$$L_c = \frac{(1-k)R}{2f}$$

critical value of the capacitor C_c

$$C_c = \frac{K}{2fR}$$

Advantages:

1) It provides output voltage polarity reversal without a transformer.

2) High Efficiency.

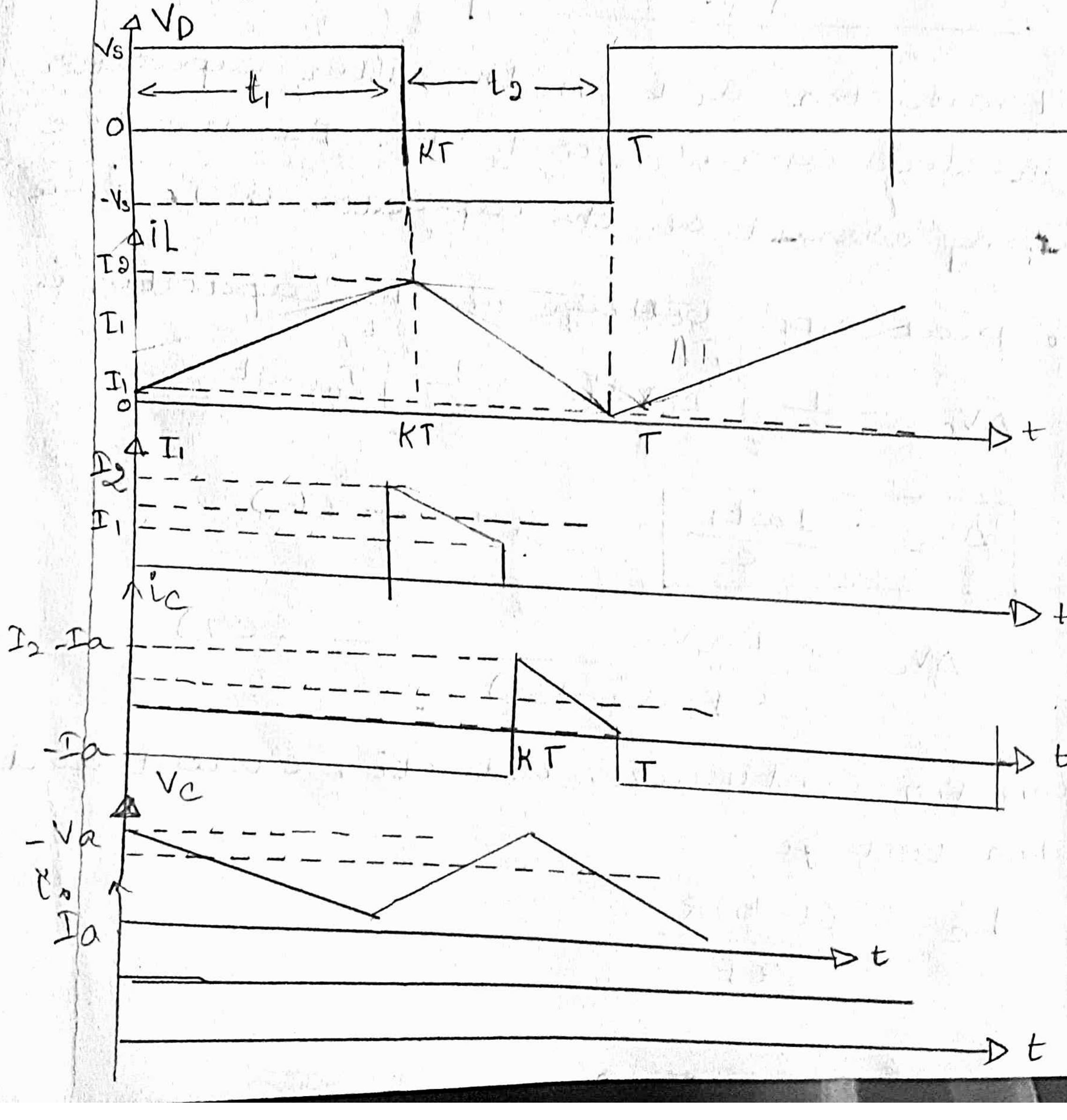
3) Under a fault condition, the di/dt of the fault current is limited by the inductor L.

4) Output short circuit protection easy to implement

Disadvantages:

1) Input current is discontinuous.

2) high peak current flows through transistor Q_1.



RESONANT CONVERTER

Generally in converters, the power devices are made to turn ON and turn OFF the entire load current with high di/dt or with high dv/dt. This increases the power losses in the switching device.

In order to minimize this effects, the power devices are turned ON and OFF, when the voltage across it or current through it is zero at the instant of switching. The converter circuits which employs zero-voltage and zero-current switching are called resonant converters.

The resonant converters are of two types:

They are
(i) zero current switching (ZCS)
(ii) zero Voltage switching (ZVS).

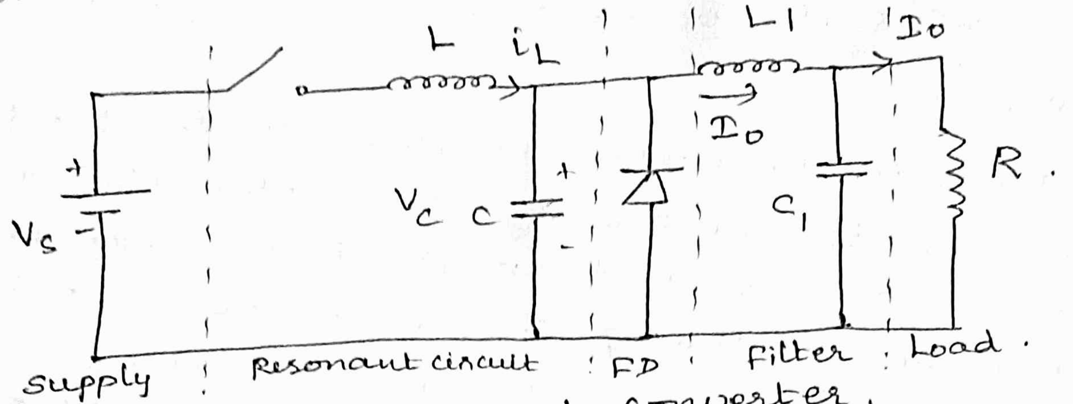
Zero current switching Resonant converters:

There are two types of ZCS resonant converters.

They are
1) L-type
2) M-type.

L type ZCS Resonant Converters:-

The L-type ZCS Resonant converter is shown in the figure. The switching device S can be any one of the power semiconductor devices like GTO, Thyristor, BJT, MOSFET. Inductor L and capacitor C near the d.c source (V_s) form a resonant circuit whereas L_1, C_1 near the load constitute a filter circuit.



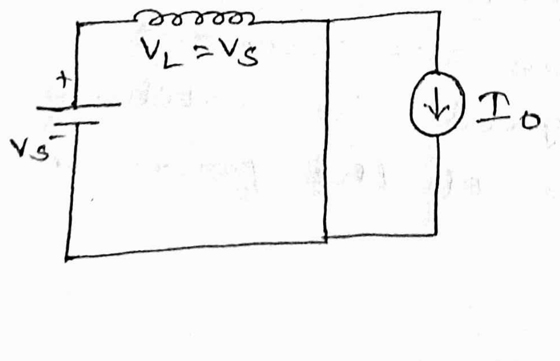
RCS Resonant converter.

The current in the output inductor L_1 is assumed to be ripple free and equal to the load current I_o.

Initially when the switch is open, the diode is forward biased, to carry the output inductor current and the voltage across C is zero.

Analysis for $0 \leq t \leq t_1$

when the switch is ON, the diode initially remains forward-biased to carry I_o and the voltage across L is the same as that of the source voltage V_s.



(i) The switch is closed at $t = 0$, the diode is ON the voltage across L is V_s. The current in L is expressed as,

$$i_L(t) = \frac{1}{L}\int V_s \cdot dt.$$

$$i_L(t) = \frac{V_s}{L}\int dt$$

$$\boxed{i_L(t) = \frac{V_s}{L}\, t} \quad \text{—— (1).}$$

at $t = t_1$, i_L reaches I_o, hence

sub $i_L(t) = I_o$
$t = t_1$ in
eqn (1).

$$i_L(t) = I_o$$

inductor
L
$V_L(t) = L \cdot di/dt$
$i_L(t) = \frac{1}{L}\int v \cdot dt$

capacitor
C
$V_c(t) = \frac{1}{C}\int i\,dt$
$i_c(t) = C \cdot \frac{dv}{dt}$

$$I_o = \frac{V_s}{L}\, t_1.$$

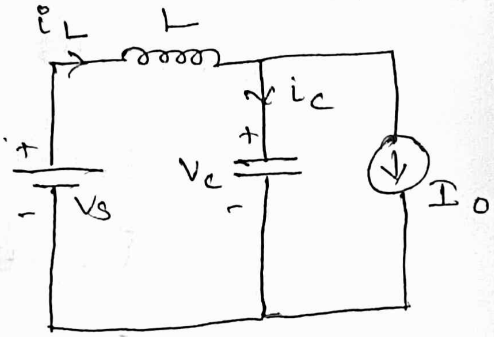
$$\boxed{t_1 = \frac{I_o \cdot L}{V_s}}$$

Analysis for $t_1 \le t \le t_2$:-

The current in L increases linearly and the diode remains forward biased. the diode turns off when i_L reaches I_o,

If I_o is a constant, the load appears as a current source and the LC circuit oscillates. Consequently i_L returns to zero and remains there. The switch is turned off after the current reaches zero, resulting in zero current switching and no switching power loss.

When the diode (D) current reduces to zero, the resonant capacitor C is charged resonantly by a current ($I_{Lz} - I_o$). ~~The inductor current is given by~~,



① ⟵ $$V_c(t) = V_s - L \cdot \frac{di_L(t)}{dt}$$

② ⟵ $$i_c(t) = i_L(t) - I_o.$$

diff ①,

$$\frac{dv_c(t)}{dt} = -L \cdot \frac{d^2 i_L(t)}{dt^2}$$

$$\frac{i_c(t)}{C} = -L \cdot \frac{d^2 i_L(t)}{dt^2}$$

$$\frac{i_L(t) - I_o}{C} = -L \cdot \frac{d^2 i_L(t)}{dt^2}$$

$$\frac{I_o - i_L(t)}{C} = L \cdot \frac{d^2 i_L(t)}{dt^2}$$

$\div LC$,

$$\frac{I_o - i_L(t)}{LC} = \frac{d^2 i_L(t)}{dt^2}$$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{i_L(t)}{LC} = \frac{I_o}{LC} \quad \text{—— ④}$$

The solution of eqn ④ with initial condition $i_L(t_1) = I_o$.

$$\boxed{i_L(t) = I_o + \frac{V_s}{Z_o} \sin \omega_o (t - t_1)} \quad \text{—— ⑤}$$

$Z_0 = \sqrt{\frac{L}{C}}$; $\omega_0 = \frac{1}{\sqrt{LC}}$.

Eqn (5) is valid until i_L reaches zero at $t = t_2$

The switch is turned off after the current reaches zero, resulting in zero current switching and no switching power loss.

$$i_L(t) = 0, \text{ at } t = t_2.$$

(5) $\Rightarrow$ $0 = I_0 + \frac{V_s}{Z_0} \sin\omega_0 (t_2 - t_1)$.

$$\sin\omega_0 (t_2 - t_1) = -\frac{I_0 Z_0}{V_S}.$$

$$t_2 - t_1 = \frac{1}{\omega_0} \sin^{-1}\left[-\frac{I_0 Z_0}{V_S}\right].$$

Sub (5) in (1).

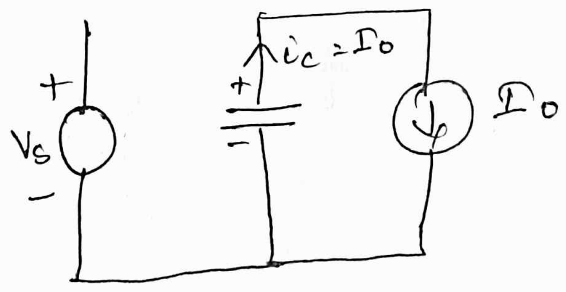
$$V_C(t) = V_S - L \cdot \frac{d}{dt}\left[I_0 + \frac{V_S}{Z_0} \sin\omega_0 (t - t_1)\right].$$

$$= V_S - L \cdot \frac{V_S}{Z_0} \omega_0 \cdot \cos\omega_0 \cdot (t - t_1).$$

$$= V_S \left[1 - \frac{L}{Z_0} \omega_0 \cdot \cos\omega_0 (t - t_1)\right].$$

$$V_C(t) = V_S \left[1 - \cos\omega_0 (t - t_1)\right].$$

Analysis for $t_2 \le t \le t_3$:-



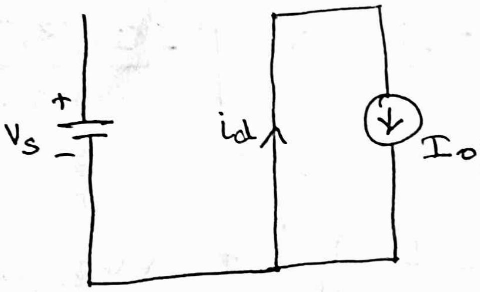
After the inductor current reaches zero at t_2, the switch is opened. The diodes is off since $V_c > 0$. capacitor current is $-I_o$.

$$V_c(t) = \frac{1}{C} \int_{t_2}^{t} -I_o \, dt + V_c(t_2).$$

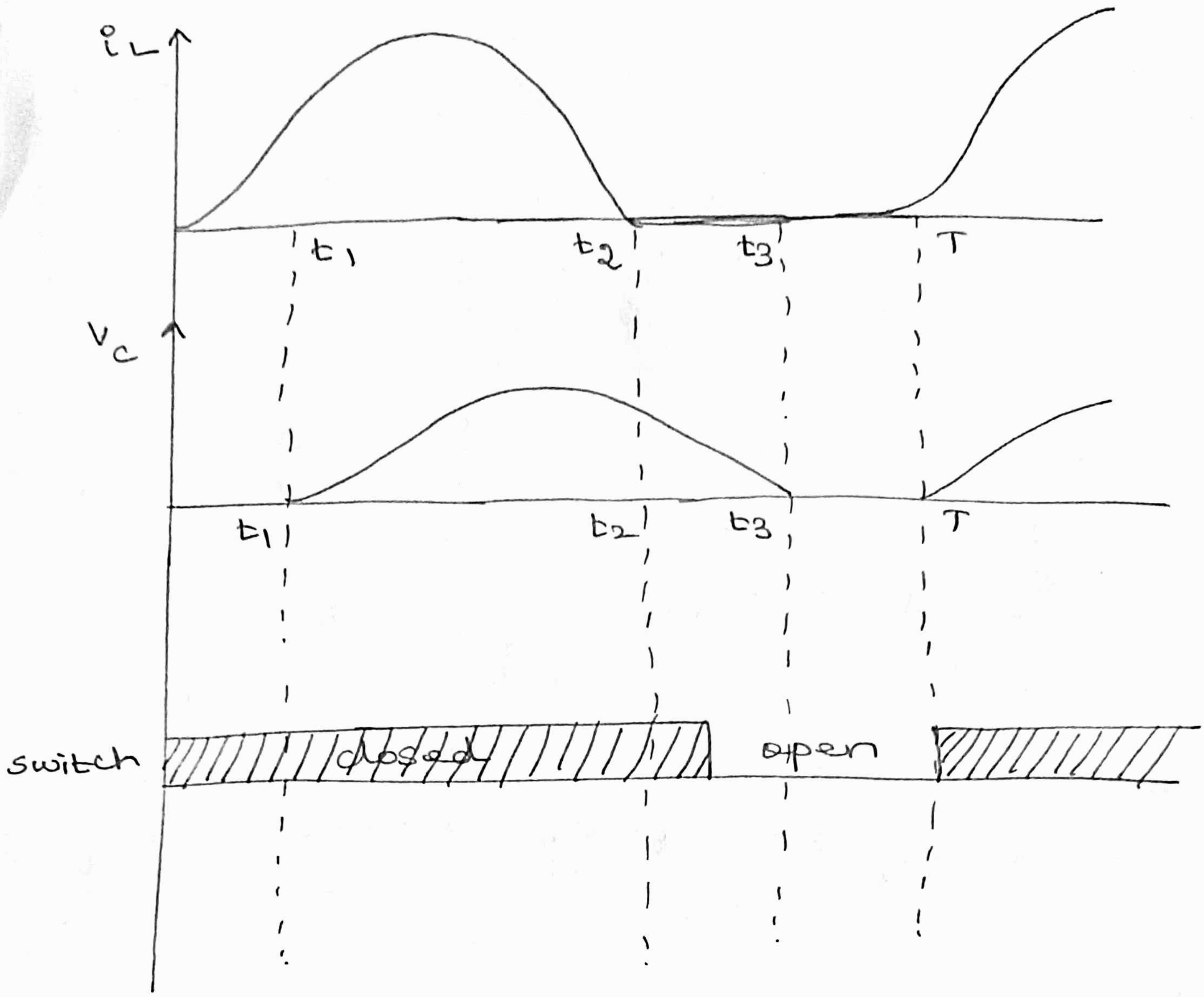
$$= -\frac{I_o}{C}(t_2 - t) + V_c(t_2).$$

$$V_c(t) = \frac{I_o}{C}(t - t_2) + V_c(t_2).$$

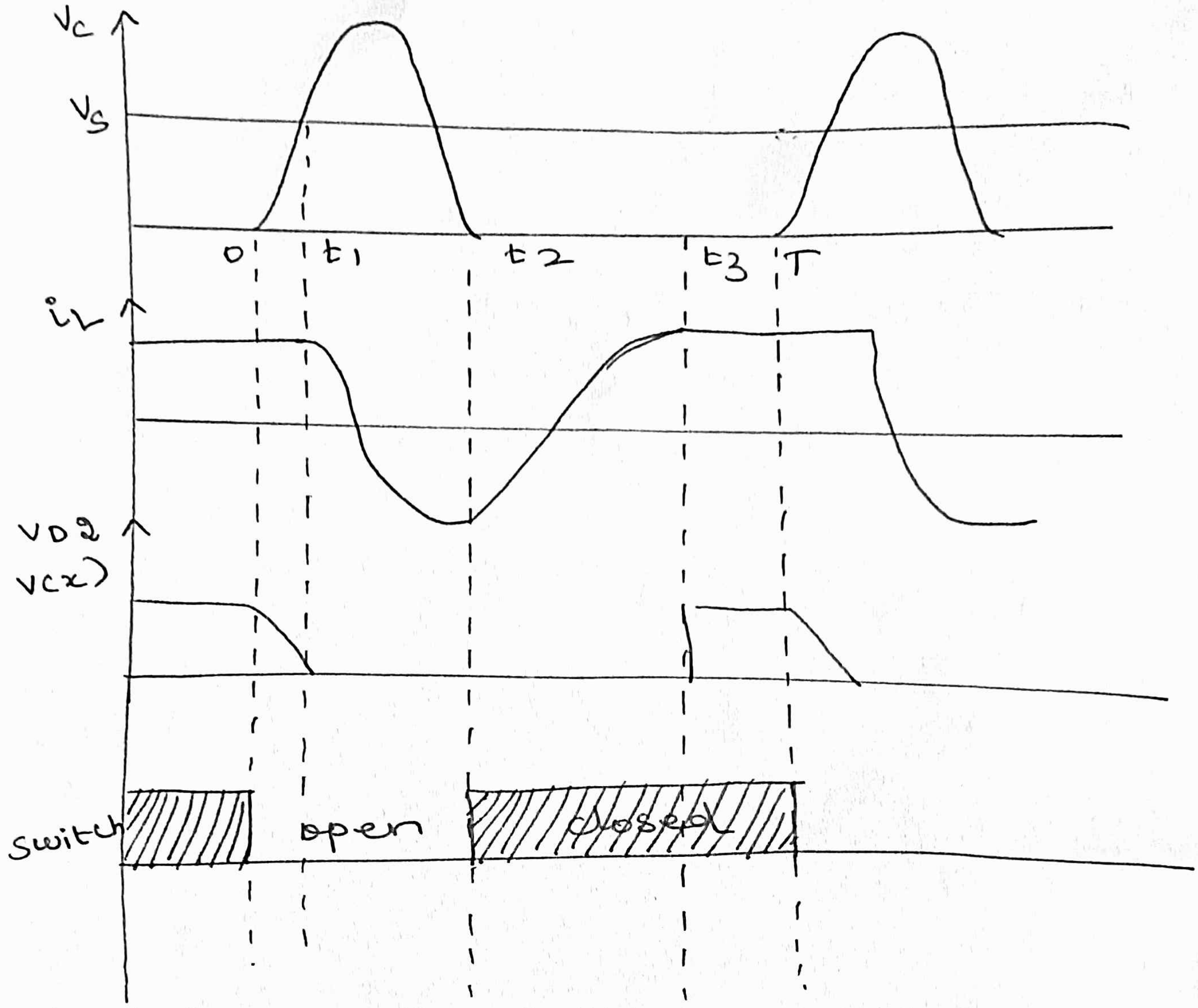
Analysis for $t_3 \le t \le T$:



If I_o is constant, the capacitor voltage decreases linearly. when the capacitor voltage reaches zero, the diode becomes forward biased to carry I_o. The circuit is then back at the starting point. The duration of this interval is the difference between the switching period T and the other time intervals which are determined from other circuit parameters.



ZCS → waveform.

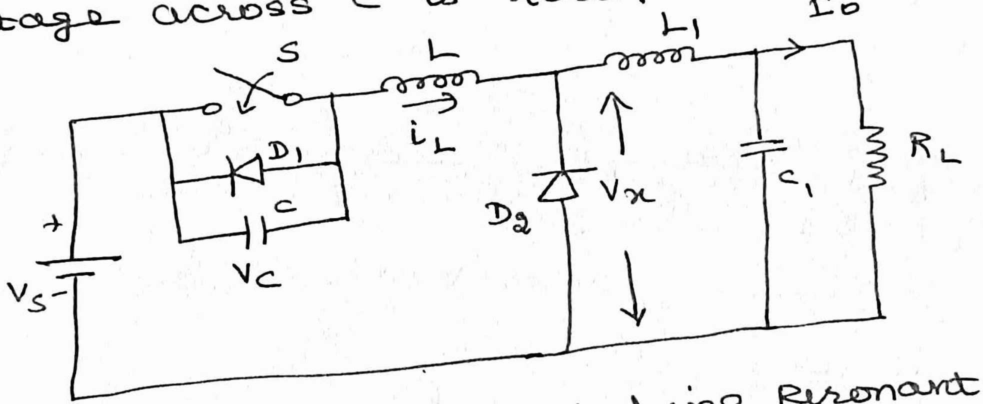


ZVS → waveform.

Zero Voltage Switching Resonant Converter

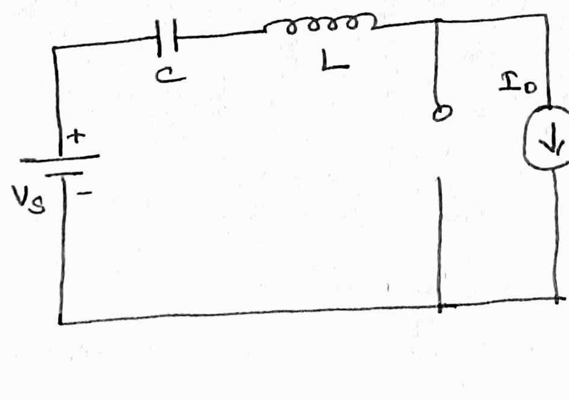
The zero voltage switching (ZVS) resonant converter is shown in figure. It consists of diode D_1, capacitor C connected across the switch S. It has L, C as the resonant circuit components and L_1, C_1 as the filter circuit components.

The analysis assumes that the output filter produces a ripple free current I_o. Beginning with the switch closed, the current in the switch and the L is I_o, the current in D_1 & D_2 are zero and the voltage across C is zero.



Zero Voltage switching Resonant converter

Analysis of $0 \le t \le t_1$



The switch is opened at $t=0$, The capacitor current is then I_o, causing the capacitor voltage initially zero to increase linearly. The voltage across the capacitor c is,

$$V_c(t) = \frac{1}{c} \int I_o . dt .$$

$$\boxed{V_c(t) = \frac{I_o t}{c}} \quad \text{—— (1)}$$

The voltage across L is zero. The voltage at the filter input is

$$V_x(t) = V_s - V_c(t)$$

$$\boxed{V_x(t) = V_s - \frac{I_o t}{c}} \quad \text{—— (2)}$$

At $t = t_1$, $V_x = 0$, the diode turns ON.

$$0 = V_s - \frac{I_o t_1}{c} .$$

$$\boxed{t_1 = \frac{V_s . c}{I_o}} \quad \text{—— (3)} \qquad c = \frac{t_1 I_o}{V_s} .$$

Eqn ② can be expressed as,

$$V_x(t) = V_s - \frac{\cancel{I_o} t}{\frac{\cancel{I_o} t_1}{V_s}}$$

sub c value in eqn ②.

$$= V_s - V_s \times t/t_1$$

$$\boxed{V_x(t) = V_s \left[1 - t/t_1\right]} \quad \text{—— (4)} .$$

Analysis for $t_1 \le t \le t_2$

When V_c reaches the source voltage V_s the diode D_2 becomes forward biased. At this time i_L and V_c in the circuit begin to oscillate.

$$V_c(t) = V_s + I_0 Z_0 \sin[\omega_0 (t - t_1)] \quad — ⑧$$

$$Z_0 = \sqrt{\frac{L_1}{C_1}}$$

At $t = t_2$, $V_c = 0$ from eqn ⑧

$$0 = V_s + I_0 Z_0 \sin[\omega_0 (t_2 - t_1)]$$

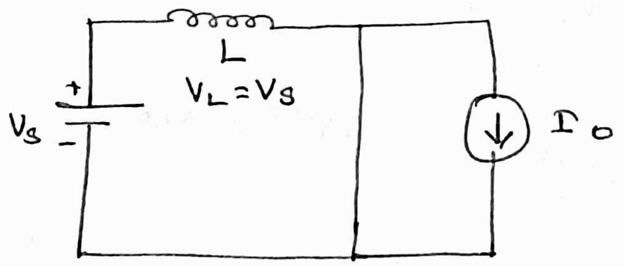
$$I_0 Z_0 \sin[\omega_0 (t_2 - t_1)] = -V_s$$

$$\sin[\omega_0 (t_2 - t_1)] = \frac{-V_s}{I_0 Z_0}$$

$$\omega_0 (t_2 - t_1) = \sin^{-1}\left[\frac{-V_s}{I_0 Z_0}\right]$$

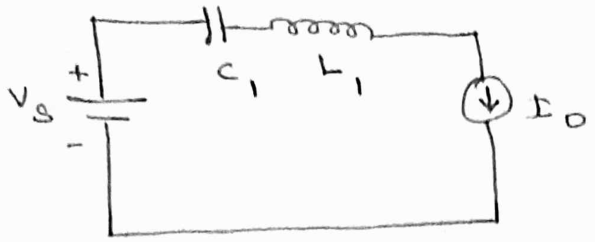
$$\boxed{t_2 = \frac{1}{\omega_0}\left[\sin^{-1}\left(-\frac{V_s}{I_0 Z_0}\right)\right] + t_1} \quad — ⑨$$

Analysis of $t_2 \leq t \leq t_3$



After t_2, both diodes are forward biased, the voltage across L_1 is V_s, i_L increases linearly until it reaches I_0 at t_3. The switch is closed after t_2 when $V_c = 0$ and the diode is ON to carry a negative current i_L. The current $i_L(t)$ in the interval from t_2 to t_3 is expressed as,

$$i_L(t) = \frac{1}{L_1}\int_{t_2}^{t} V_s \, dt + i_L(t_2).$$



$$L_1 \frac{di_L(t)}{dt} + V_{c_1}(t) = V_s.$$

Diff,

$$L_1 \frac{d^2 i_L(t)}{dt^2} + \frac{dV_{c_1}(t)}{dt} = 0 \quad \text{——— (5)}$$

$$\text{but } \frac{dV_{c_1}(t)}{dt} = \frac{i_c(t)}{C_1} \quad \text{——— (6).}$$

sub (6) in (5),

$$L_1 \frac{d^2 i_L(t)}{dt^2} + \frac{i_c(t)}{C_1} = 0.$$

$i_c(t) = i_L(t)$ since both are in series.

$$\frac{d^2 i_L(t)}{dt^2} + \frac{i_L(t)}{L_1 C_1} = 0.$$

solving for $i_L(t)$ with initial condition,

$$i_L(t_1) = I_o.$$

$$\boxed{i_L(t) = I_o \cos[\omega_o(t - t_1)]} \quad \text{——— (7).}$$

$$\omega_o = \frac{1}{\sqrt{L_1 C_1}}.$$

capacitor voltage is,

$$V_c(t) = \frac{1}{C_1} \int_{t_1}^{t} i_c(t).dt + V_c(t_1).$$

$$= \frac{1}{C_1} \int_{t_1}^{t} I_o \cos[\omega_o(t - t_1)]\, dt + V_s$$

$$i_L(t) = \frac{V_s}{L_1}(t - t_2) + I_0 \cos[\omega_0 (t_2 - t_1)] \quad \text{--- (10)}$$

current at t_3 ~~in~~ is I_0,

$$i_L(t_3) = I_0.$$

$$I_0 = \frac{V_s}{L_1}(t_3 - t_2) + I_0 \cos[\omega_0 (t_2 - t_1)]$$

$$t_3 = \frac{L_1 I_0}{V_s}[1 - \cos(\omega_0 (t_2 - t_1))] + t_2.$$

At $t = t_3$, diode D_2 turns on,

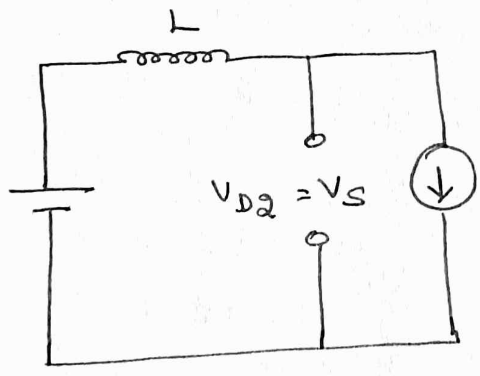
hence $V_x = 0$.

Analysis for $t_3 \leq t \leq T$

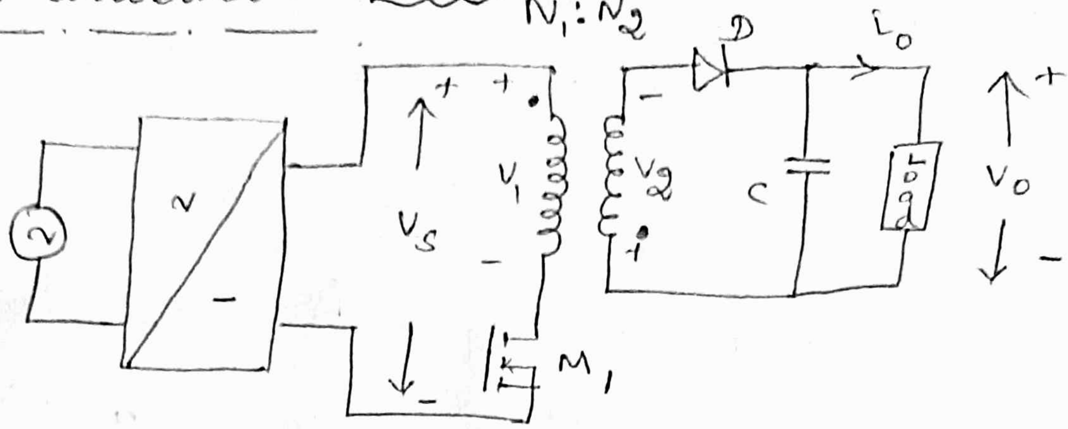
In this interval, the switch is closed both diodes are OFF, the current in the switch is I_0. The circuit in this condition until the switch is reopened.

$$\boxed{V_x = V_s}$$

The time interval $T - t_3$ is determined by the switching frequency of the circuit.



Fly Back converter :-



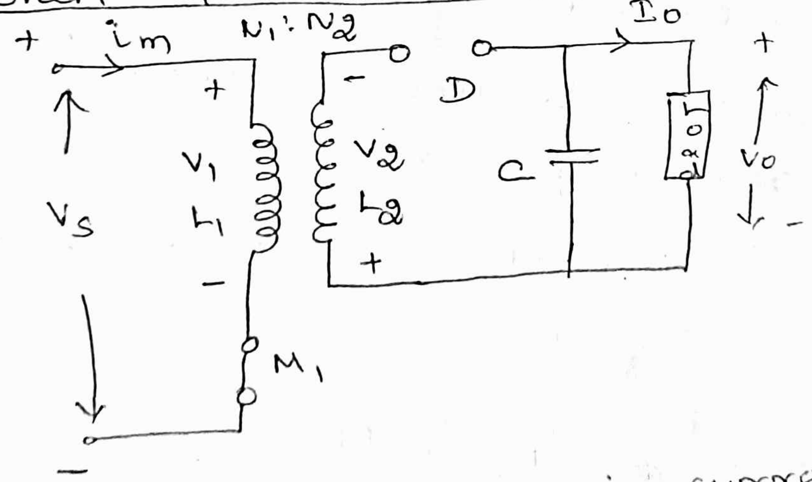
⇒ It consists of a power MOSFET M_1, transformer for isolation purposes, diode C, capacitor C and load.

⇒ when power MOSFET is turned on, supply voltage V_s is applied to the transformer ~~secondary~~ Primary

$V_1 = V_s$, D → reverse biased.

⇒ A corresponding voltage V_2 induced in the secondary. $V_2 = \frac{V_s}{N_1} \times N_2$.

when M_1 is turned ON :



Transformer magnetizing current at $t = 0$, is not zero, but has some positive value I_{mo}.

The differential ...

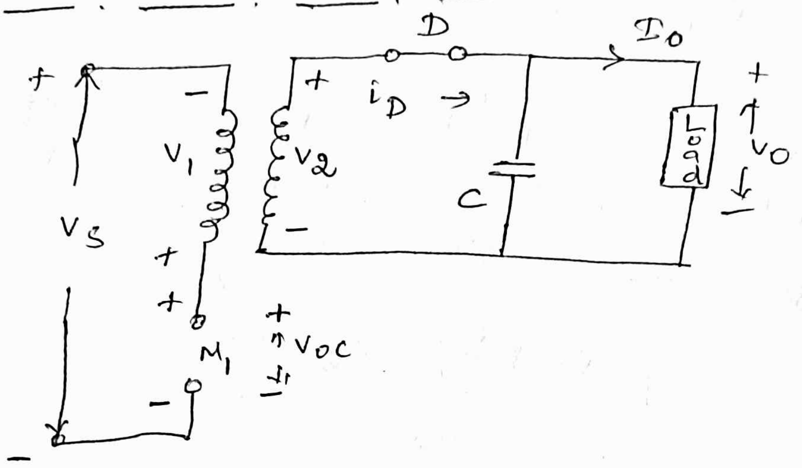
During T_{ON}, magnetizing current rises linearly from its initial value I_{mo} to I_{m1} at $t = T_{ON}$.

$$i_m(t) = I_{mo} + \frac{V_s}{L} t \quad \ldots \quad 0 < t < T_{ON}.$$

$L \Rightarrow$ Transformer magnetizing inductance (H).

at $t = T_{ON}$, $i_m(T_{on}) = I_{m1} = I_{mo} + \frac{V_s}{L} T_{ON}$.

When M_1 is turned off :-



$\Rightarrow$ The emfs induced in primary and secondary windings are reversed. Diode D is forward biased.

$\Rightarrow$ A current in transformer secondary winding begins to flow through D.

$\Rightarrow$ magnetizing current i_m reduces from I_{m1} to I_{mo} at $t = T$.

- During T_{off} $M_1 = $ Off.

$V_2 = -V_o$.

referred to primary is $V_1 = -\frac{V_o}{N_2} \times N_1$

The fall of current i_m during T_{off} can be expressed as under : —— (I).

$$i_m(t) = I_{m1} - \frac{V_o}{N_2} N_1 \times \frac{1}{L}(t - T_{on}) \qquad T_{on} < t < T.$$

at $t = T$,

$$i_m(T) = I_{m1} - V_o \times \frac{N_1}{N_2} \times \frac{1}{L}(T - T_{ON}).$$

sub the value of I_{m1}

$$i_m(T) = I_{m0} + \frac{V_s}{L} T_{ON} - V_o \times \frac{N_1}{N_2} \times \frac{1}{L}(T - T_{ON}).$$

Net energy stored in core over periodic time T is zero,

$$i_m(0) = i_m(T).$$

$$i_m(0) = I_{m0} + \frac{V_s}{L} \cdot T_{ON} - V_o \times \frac{N_1}{N_2} \frac{1}{L}(T - T_{on}).$$

$$V_s \cdot T_{ON} = \frac{V_o}{a}(T - T_{ON}). \qquad a = \frac{N_2}{N_1}.$$

$$V_o = \frac{a \cdot V_s \cdot T_{ON}}{T - T_{ON}} = \frac{a \cdot V_s \cdot k}{1 - k}$$

$k = \frac{T_{ON}}{T}$ => duty cycle of flyback converter.

open circuit voltage across N_1 is,

$$V_{oc} = V_1 + V_s = V_o \times \frac{N_1}{N_2} + V_s = \frac{V_o}{a} + V_s.$$

$$V_{oc} = \frac{a \cdot V_s \cdot k}{(1-k)\,a} + V_s = \frac{V_s}{1-k}.$$

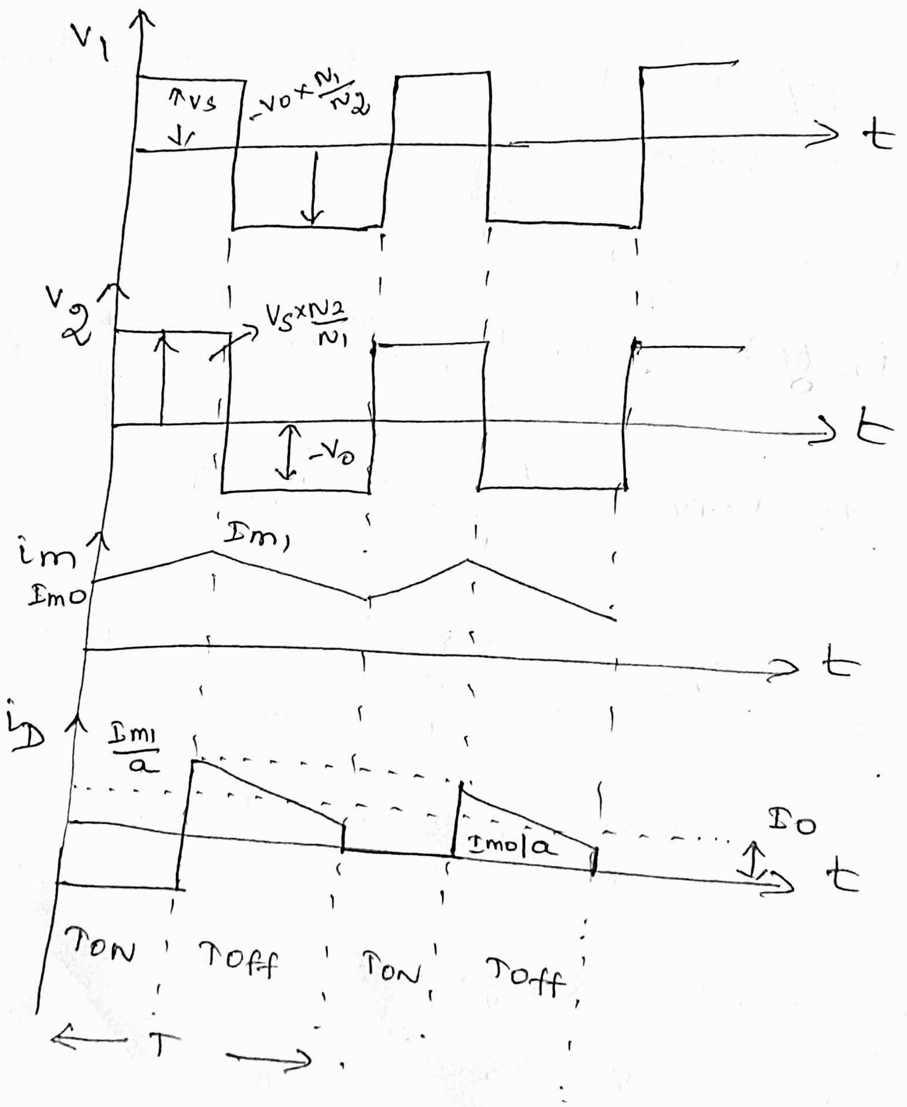
The differential ... eqn ① gives current on primary side of the transformer.

$$i_D(t) = i_m(t) \cdot \frac{N_1}{N_2}$$

$$= \frac{N_1}{N_2}\left[I_{m1} - V_o \times \frac{N_1}{N_2} \times \frac{1}{L}(t - T_{ON}) \right].$$

$$= \frac{I_{m1}}{a} - \frac{V_o}{a^2 L}(t - T_{ON}).$$

Flyback converter offers simple SMPS and is useful for applications below about 500W.



Introduction to snubber and Driver circuits :

A snubber circuit limits or stops (snubs) switching voltage amplitude and its rate of rise, therefore reducing power dissipation. A snubber circuit basically consists of a resistor and capacitor connected across the thyristor.

MOSFET Driver circuit :-

→ A driver circuit need to turn ON the semiconducting devices.

→ A MOSFET usually needs a gate driver to do the on/off operation at the desired frequency.

→ MOSFET is a voltage-driven device, no DC current flows into the gate.

→ In order to turn on a MOSFET, a voltage higher than the rated gate threshold voltage V_{th} must be applied to the gate.

→ MOSFETs are often used as switching devices at frequencies ranging from several KHZ to more than several hundreds of KHz.

→ Low power consumption needed for gate drive is an advantage of a MOSFET as a switching device.

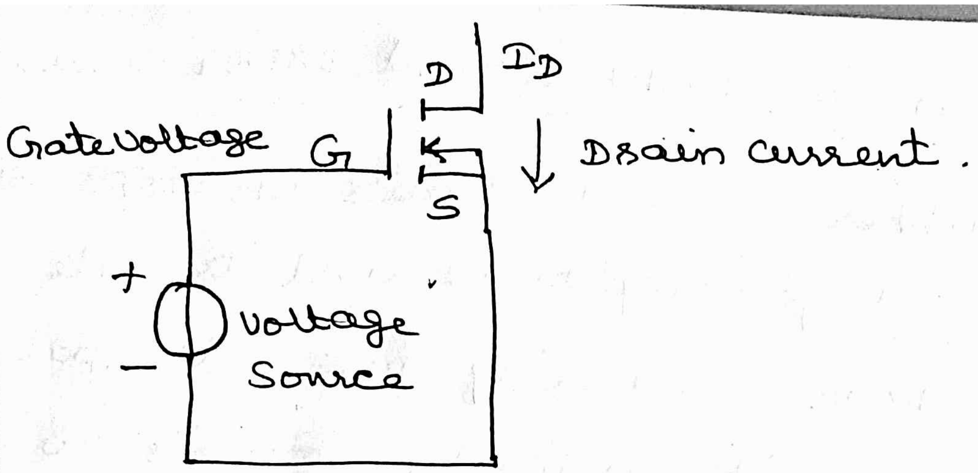


Fig: Driving a MOSFET

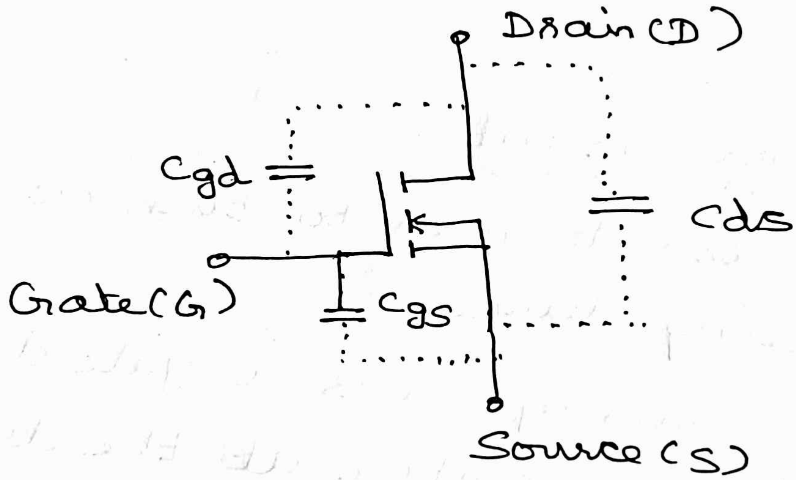


Fig: capacitances in MOSFET.

Input capacitance $= C_{gd} + C_{gs}$.

Output capacitance $= C_{ds} + C_{gd}$

Reverse Transfer capacitance $= C_{gd}$.

→ Gate voltage of a MOSFET does not increase unless its gate input capacitance is charged and the MOSFET does not turn ON until its gate voltage reaches the V_{th}.

→ V_{th} of a MOSFET is defined as the minimum gate bias required for creating a conduction channel between its source and drain regions.

Snubber Circuit :-

Snubber circuits provide protection against transient voltages that occur during turn-off. a simple RC snubber uses a resistor R in Series with a capacitor C. The RC Circuit is connected in parallel with a power MOSFET.

cutting off a current in a circuit causes a voltage to increase sharply due to stray inductance. This snubber damps this surge voltage to protect the power MOSFET.

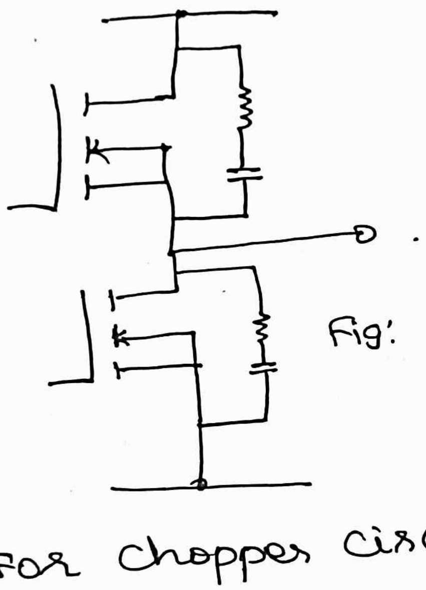


Fig: RC Snubber.

→ Ideal for chopper circuits

→ Power loss caused by the RC snubber resistor is so large, and not suitable for high frequency switching applications.

→ Power P dissipated by the snubber resistor is calculated as follows :

$$P = C_S \cdot Ed^2 \cdot f.$$

→ The capacitor acts as charge storage and the resistor path provides a discharge path.

→ The RC snubber R_1, C_1 protects the MOSFET Q_1 from voltage spike on the drain.

→ When the MOSFET is off snubber capacitor will charge through R_1.

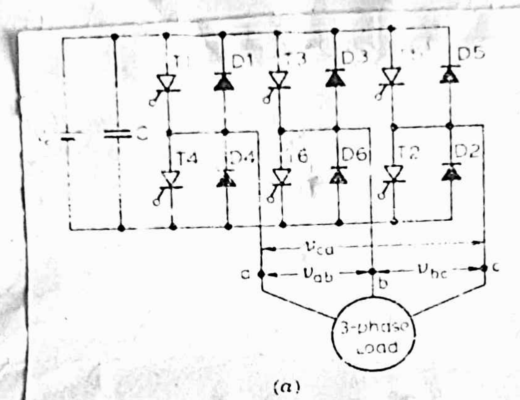
→ When the MOSFET is ON, the capacitor will discharge through R_1 to the MOSFET and to the circuit ground.

UNIT - IV

Inverters.

Single phase and three phase voltage source inverters (both 120° mode and 180° mode) - Voltage & harmonic control - PWM techniques : sinusoidal PWM, modified sinusoidal PWM - multiple PWM - introduction to space vector modulation - current source inverter.

Three phase voltage source inverter (180° mode) :-



(a)

Three phase inverter is a six step bridge inverter. It uses a minimum of 6 thyristor. A step is defined as a change in the firing from one thyristor to the next thyristor in proper sequence. A large capacitor is used to make the input dc voltage constant.

T_1 triggered at $\omega t = 0°$, and conducts for $180°$.

T_2 triggered at $\omega t = 60°$, and conducts for $60° + 180° = 240°$.

T_3 triggered at $\omega t = 120°$, conducts for $120° + 180° = 300°$

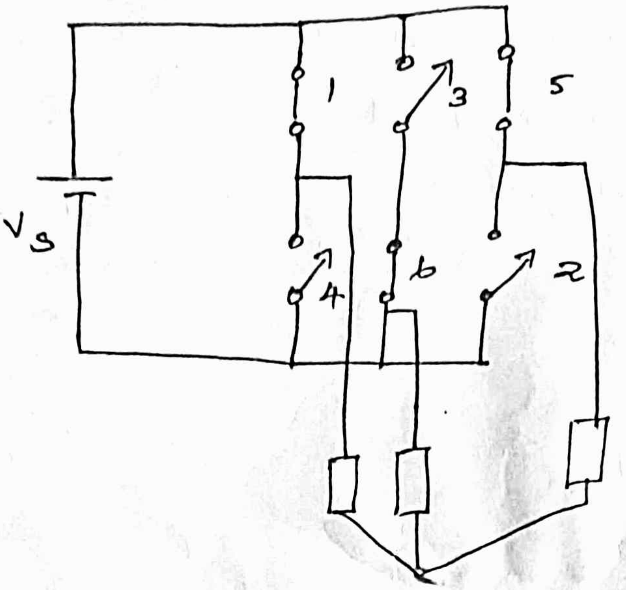
T_4 triggered at $\omega t = 180°$, conducts for $180° + 180° = 360°$

T_5 triggered at $\omega t = 240°$, conducts for $240° + 180° = \cancel{300} \; 420°$.

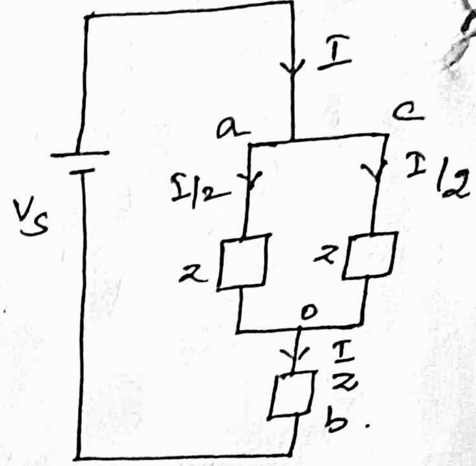
T_6 triggered at $\omega t = 300°$, conducts for $300° + 180° = 480°$.

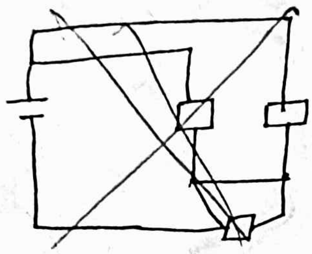
Equivalent circuit :

Mode I : $0-60^\circ$, 5, 6, 1 are conduct



$\Rightarrow$





Total impedance,

$$Z \parallel Z + Z = \frac{Z \times Z}{Z+Z} + Z$$

$$= \frac{Z^2}{2Z} + Z$$

$$= \frac{3Z^2}{2Z}$$

$$\boxed{Z_{equ} = \frac{3Z}{2}}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\frac{3Z}{2}} = \frac{2V_s}{3Z}$$

$$V_{ao} = I/2 \times Z = \frac{\cancel{2}V_s}{\cancel{2} \times \cancel{3}Z} \times \frac{\cancel{3Z}}{\cancel{2}}$$

$$= \frac{V_s}{3}.$$

$$V_{bo} = -I \times Z = -\frac{2V_s}{3Z} \times Z = -\frac{2V_s}{3}.$$

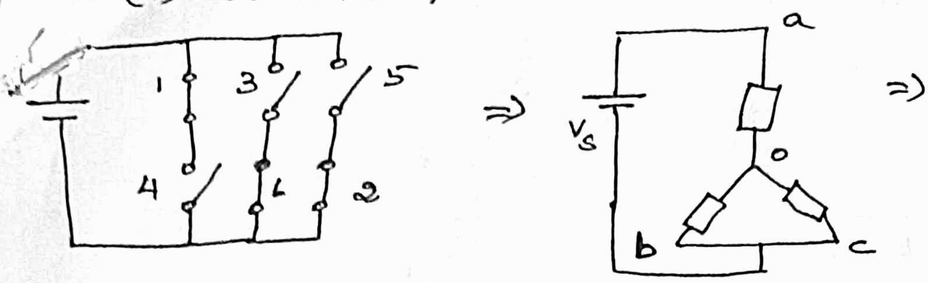
$$V_{co} = I/2 \times Z = \frac{2V_s}{3Z} \times Z/2 = V_s/3.$$

$$V_{ab} = V_{ao} + V_{ob} = V_s/3 + \frac{2V_s}{3} = V_s.$$

$$V_{bc} = V_{bo} + V_{oc} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s.$$

$$V_{ca} = V_{co} + V_{oa} = V_s/3 - V_s/3 = 0.$$

Mode(ii) $60° - 120°$, 6, 1, 2 are conduct



$$\Rightarrow \quad V_{ao} = \frac{2V_s}{3}$$

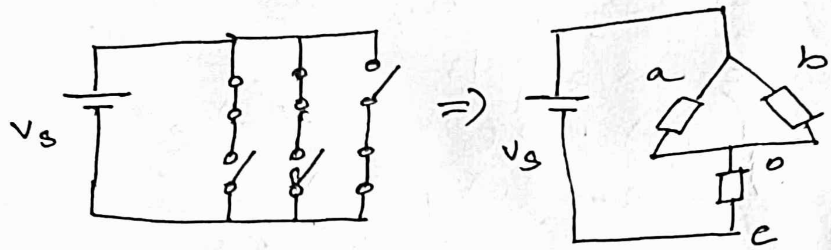
$$V_{ob} = \frac{V_s}{3}.$$

$$V_{oc} = V_s/3.$$

$$V_{ab} = V_{ao} + V_{ob} = V_s \;;\; V_{bc} = V_{bo} + V_{oc} = -\frac{V_s}{3} + \frac{V_s}{3} = 0.$$

$$V_{ca} = V_{co} + V_{oa} = -\frac{V_s}{3} - \frac{2V_s}{3} = -V_s.$$

Mode (iii) $120° - 180°$, 1, 2, 3 are conduct:



$$\Rightarrow \quad V_{ao} = \frac{V_s}{3}.$$

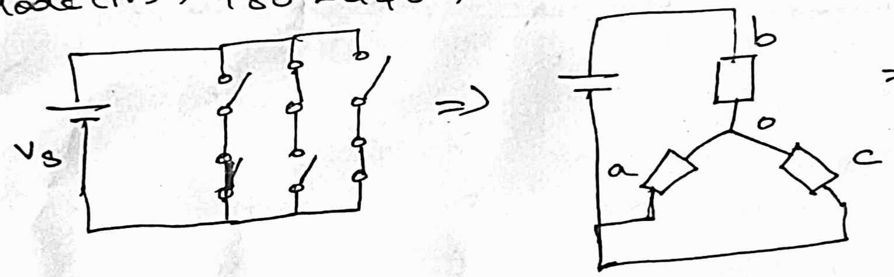
$$V_{bo} = \frac{V_s}{3}$$

$$V_{oc} = +\frac{2V_s}{3}.$$

$$V_{ab} = V_{ao} + V_{ob} = 0 \;;\; V_{bc} = V_{bo} + V_{oc} = V_s \;;$$

$$V_{ca} = V_{co} + V_{oa} = -V_s.$$

Mode (iv) ; $180° - 240°$, 2, 3, 4 conduct.



$$\Rightarrow \quad V_{bo} = \frac{2V_s}{3}$$

$$V_{oa} = V_s/3.$$

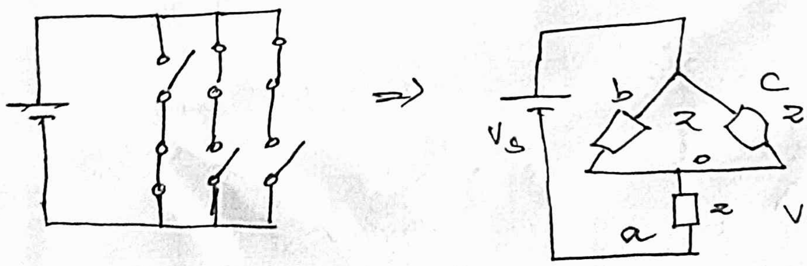
$$V_{oc} = V_s/3.$$

$$V_{ab} = V_{ao} + V_{ob} = -\frac{V_s}{3} - \frac{2V_s}{3} = -V_s.$$

$$V_{bc} = V_{bo} + V_{oc} = \frac{2V_s}{3} + V_s/3 = V_s.$$

$$V_{ca} = V_{co} + V_{oa} = -V_s/3 + V_s/3 = 0.$$

Mode (v) ; $240° - 300°$, 3, 4, 5 are conduct



$$\Rightarrow \quad V_{bo} = V_s/3.$$

$$V_{co} = V_s/3.$$

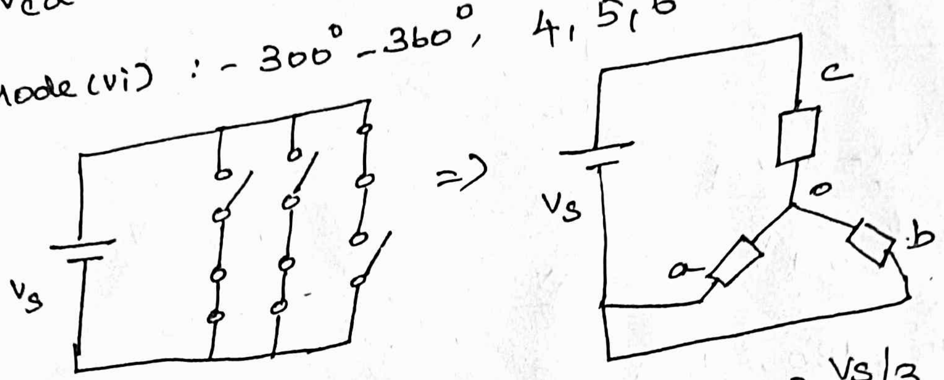
$$V_{oa} = \frac{2V_s}{3}.$$

$$V_{ab} = V_{ao} + V_{ob} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s.$$

$V_{bc} = V_{bo} + V_{oc} = \frac{V_s}{3} + \frac{2V_s}{3} = V_s$.

$V_{ca} = V_{co} + V_{oa} = \frac{V_s}{3}$

Mode (vi) :- $300° - 360°$, 4, 5, 6 are conduct.

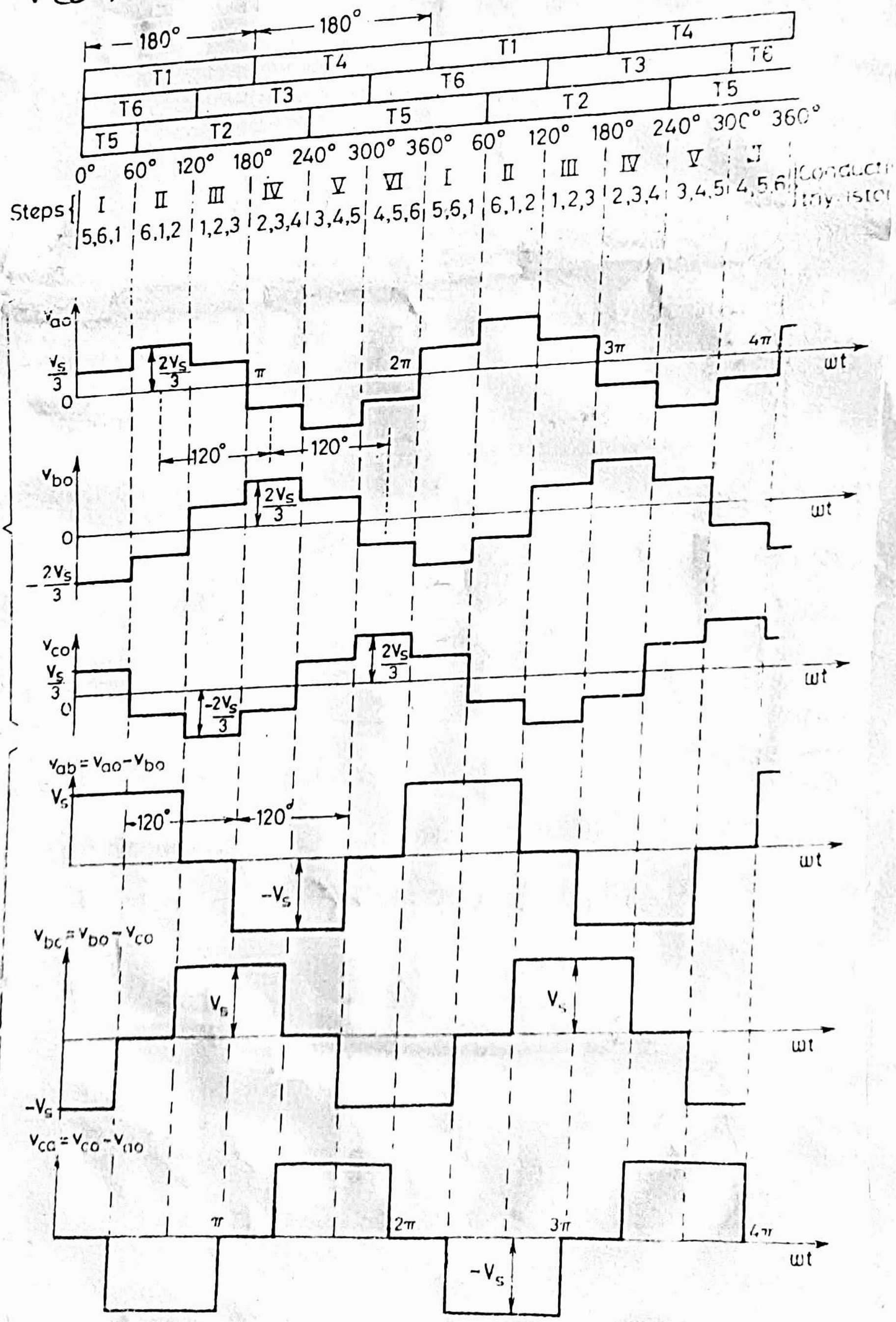


$V_{co} = \frac{2V_s}{3}$; $V_{oa} = \frac{V_s}{3}$; $V_{ob} = V_s/3$.

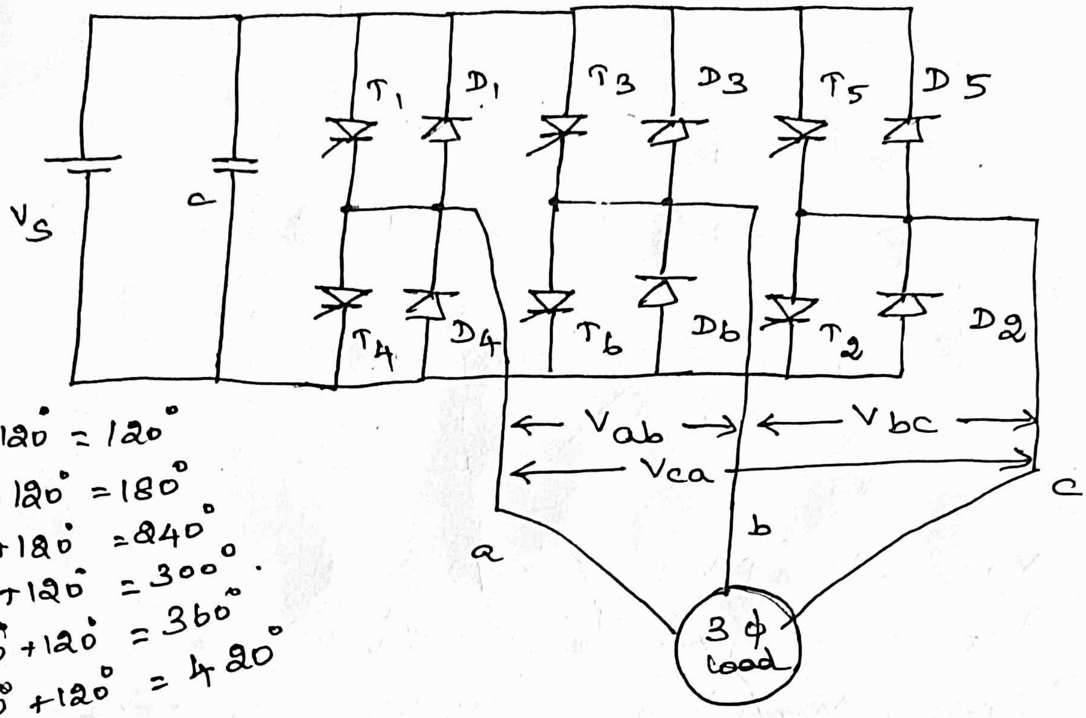
$V_{ab} = V_{ao} + V_{ob} = -V_s/3 + V_s/3 = 0$;

$V_{bc} = V_{bo} + V_{oc} = -V_s/3 - 2V_s/3 = -V_s$.

$V_{ca} = V_{co} + V_{oa} = 2V_s/3 + V_s/3 = V_s$.



120° mode with star connected inverter (3 φ).



$T_1 = 0° + 120° = 120°$

$T_2 = 60° + 120° = 180°$

$T_3 = 120° + 120° = 240°$

$T_4 = 180° + 120° = 300°$

$T_5 = 240° + 120° = 360°$

$T_6 = 300° + 120° = 420°$

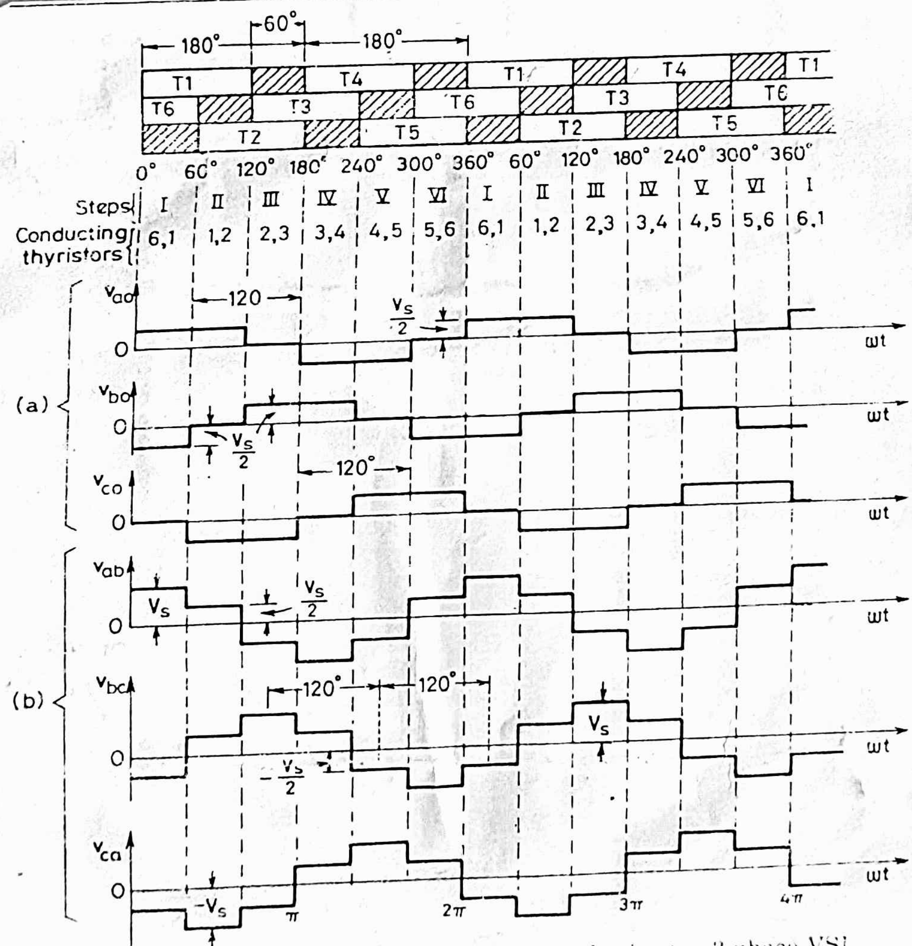
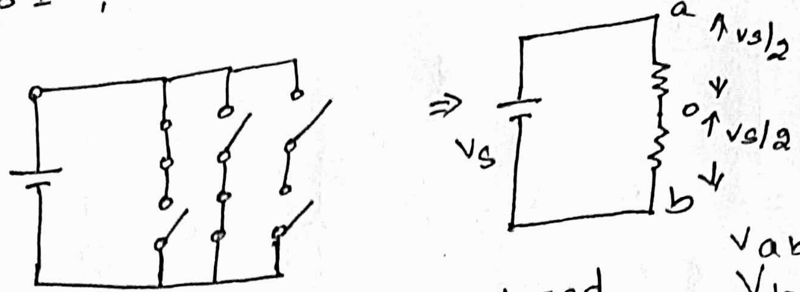


Fig. 8.22. Voltage waveforms for 120° mode six-step 3-phase VSI.

V.

Step I ; 0–60°, 6, 1 closed



$V_{ao} = V_s/2$

$V_{ob} = V_s/2$

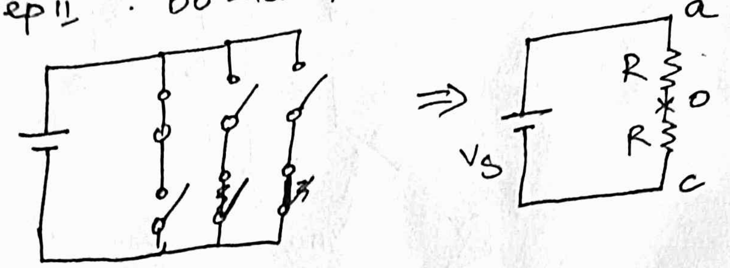
$V_{oc} = 0$

$V_{ab} = V_{ao} + V_{ob}$

$V_{bc} = V_{bo} + V_{oc}$

$V_{ca} = V_{co} + V_{oa}$

Step II : 60°–120°, 1, 2 closed



$V_{ao} = V_s/2$

$V_{oc} = V_s/2$.

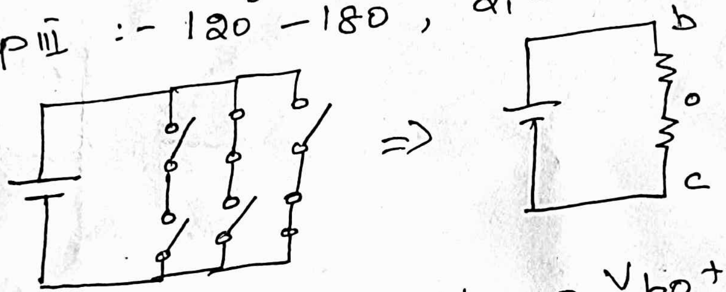
$V_{bo} = 0$.

$V_{ab} = V_{ao} + V_{ob} = V_s/2$.

$V_{bc} = V_{bo} + V_{oc} = 0 + V_s/2 = V_s/2$

$V_{ca} = V_{co} + V_{oa} = -V_s/2 - V_s/2 = -V_s$.

Step III :- 120° – 180°, 2, 3 closed.



$\Rightarrow$ $V_{ao} = 0$.

$V_{bo} = V_s/2$

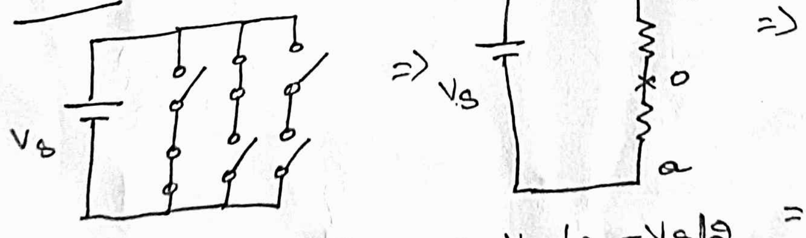
$V_{oc} = V_s/2$.

$V_{ca} = V_{co} + V_{oa} = -V_s/2$

$V_{bc} = V_{bo} + V_{oc} = V_s/2 + V_s/2 = V_s$

$V_{ab} = V_{ao} + V_{ob} = -V_s/2$

Step IV :- 180° – 240°, 3, 4 closed.



$\Rightarrow$ $V_{bo} = V_s/2$.

$V_{oa} = V_s/2$

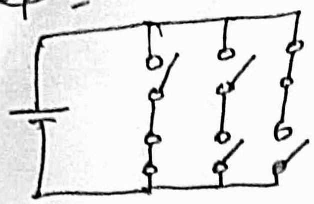
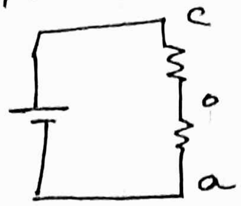
$V_{oc} = 0$.

$V_{ab} = V_{ao} + V_{ob} = -V_s/2 - V_s/2 = -V_s$.

$V_{bc} = V_{bo} + V_{oc} = V_s/2$

$V_{ca} = V_{co} + V_{oa} = 0 + V_s/2 = V_s/2$.

Step V : 240° - 300°, 4, 5 closed.

 =>  =

$V_{co} = V_s/2$

$V_{oa} = V_s/2$

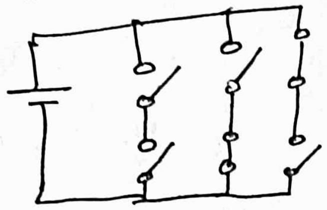
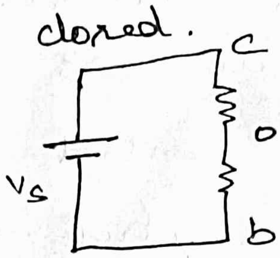
$V_{ob} = 0$.

$V_{ab} = V_{ao} + V_{ob} = -V_s/2$

$V_{bc} = V_{bo} + V_{oc} = 0 - V_s/2 = -V_s/2$

$V_{ca} = V_{co} + V_{oa} = V_s/2 + V_s/2 = V_s$.

Step VI : 300° - 360°, 5, 6 closed.

 =>  =

$V_{co} = V_s/2$

$V_{ob} = V_s/2$.

$V_{ao} = 0$.

$V_{ab} = V_{ao} + V_{ob} = V_s/2$; $V_{bc} = V_{bo} + V_{oc} = -V_s/2 - V_s/2 = -V_s$

$V_{ca} = V_{co} + V_{oa} = V_s/2 + 0 = V_s/2$.

Fourier analysis of phase voltage waveform,

$$V_{ao} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n(\omega t + \pi/6).$$

$$V_{bo} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n(\omega t - \pi/2).$$

$$V_{co} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n(\omega t + 5\pi/6).$$

$$V_{ab} = \sum_{n=6k\pm1}^{\infty} \frac{3V_s}{n\pi} \sin n\left(\omega t + \pi/3\right).$$

$k = 0, 1, 2, 3 \ldots$

Voltage control in 1ϕ inverter :-

An ac load may require a constant voltage. Any variations in the dc input voltage must be compensated in order to maintain constant voltage at the a.c load terminals.

The various methods for the control of voltage of inverters are as

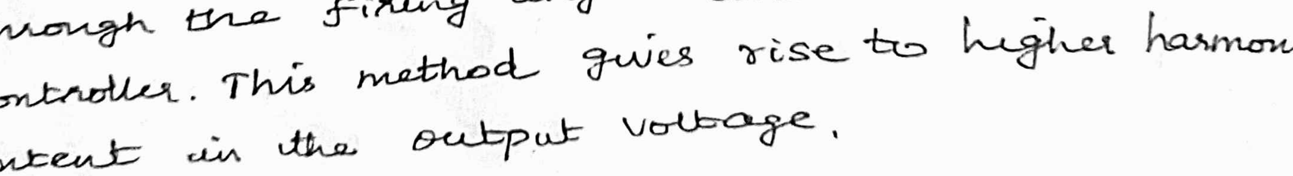
(i) External control of ac output voltage
(ii) External control of dc input voltage
(iii) Internal control of inverter.

External control of a.c output voltage :-

There are two possible methods: They are

(i) AC voltage control
(ii) series - inverter control.

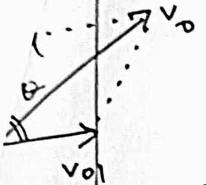
(i) AC voltage control :-



The voltage input to ac load is regulated through the firing angle control of ac voltage controller. This method gives rise to higher harmonic content in the output voltage.

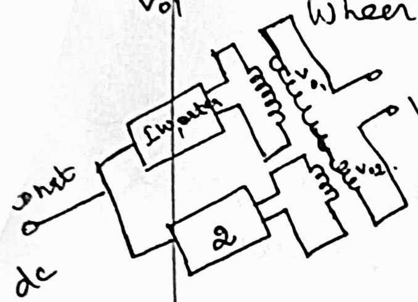
2) Series - Inverter control :-

In this method, the inverter output is fed to two transformers whose secondaries are connected in series. Phasor sum of the two fundamental voltages V_{01}, V_{02} gives the resultant fundamental voltage V_0. Here V_0 is given by,



$$V_0 = \left[V_{01}^2 + V_{02}^2 + 2V_{01}\cdot V_{02}\cdot \cos\theta\right]^{1/2}.$$

When θ is zero,



$$= \left[V_{01}^2 + V_{02}^2 + 2V_{01}\cdot V_{02}\right]^{1/2} \qquad \therefore [\cos 0 = 1]$$

$$= \left[(V_{01} + V_{02})^2\right]^{1/2}$$

$$= V_{01} + V_{02}.$$

when $\theta = \pi$, $= V_0 = 0$. in case $V_{01} = V_{02}$.

The angle θ can be varied by the firing angle control of two inverters.

(2) External control of dc input voltage :-

constant ac voltage → Fully controlled rectifier → filter → (controlled dc voltage) → Inverter → Controlled ac voltage.

constant ac voltage → uncontrolled rectifier → chopper → filter → (controlled dc) → inverter → Controlled ac voltage

constant ac voltage → Ac voltage Controller → uncontrolled rectifier → filter → (controlled dc voltage) → inverter → Controlled ac voltage.

constant dc voltage → Chopper → filter → (controlled dc voltage) → Inverter → Controlled ac voltage.

tage control of single phase inverter :-

i) Single pulse width modulation.

ii) Multiple pulse width modulation.

iii) Sinusoidal pulse width modulation.

iv) Modified sinusoidal pwm.

v) phase displacement control.

The methods are applicable to 3ϕ inverter.

i) Single pulse width modulation :-

Only one pulse per half cycle and the output rms voltage is changed by varying the width of the pulse. The gating signals are generated by comparing the rectangular control signal of amplitude A_r with triangular carrier signal A_c.

$$\text{Modulation index } M = \frac{A_r}{A_c}.$$

$$\text{RMS value of output voltage } V_{or} = \left[\frac{1}{\pi}\int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_s^2 \, d(\omega t)\right]^{1/2}$$

Fourier series of output voltage,

$$V_o = \sum_{1,3,5} (A_n \cos n\omega t + B_n \sin n\omega t).$$

Half wave symmetry, $a_o = a_n = 0$.

$$B_n = \frac{2}{\pi}\int_0^{\pi} V_s . \sin n\omega t . d\omega t = \frac{2}{\pi}\int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_s \sin \omega t . d\omega t.$$

$$= \frac{2V_s}{\pi}\left(\frac{-\cos n\omega t}{n}\right)_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} = \frac{2V_s}{n\pi}(\cos n\omega t)_{\frac{\pi+\delta}{2}}^{\pi-\delta/2}.$$

$$= \frac{2V_s}{n\pi}\left[\cos n\left(\frac{\pi-\delta}{2}\right) - \cos n\left(\frac{\pi+\delta}{2}\right)\right].$$

where $T_s - T_1 - T_2 = T$

$$V_o = \sum_{1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin\frac{n\delta}{2} \sin n\omega t$$

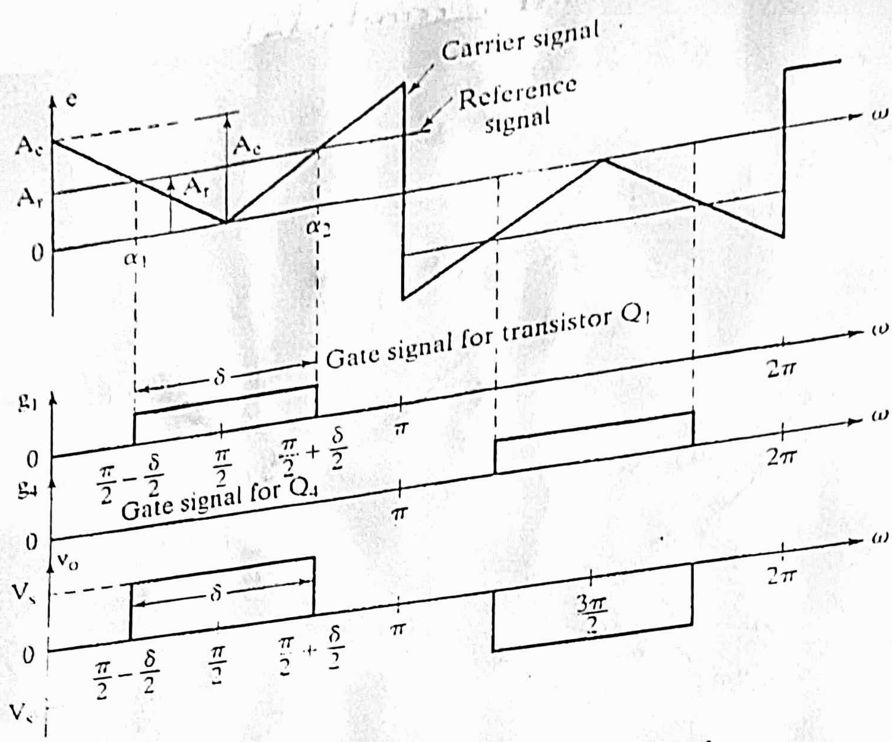


Fig 1.7 Single-Pulse-Width-Modulation

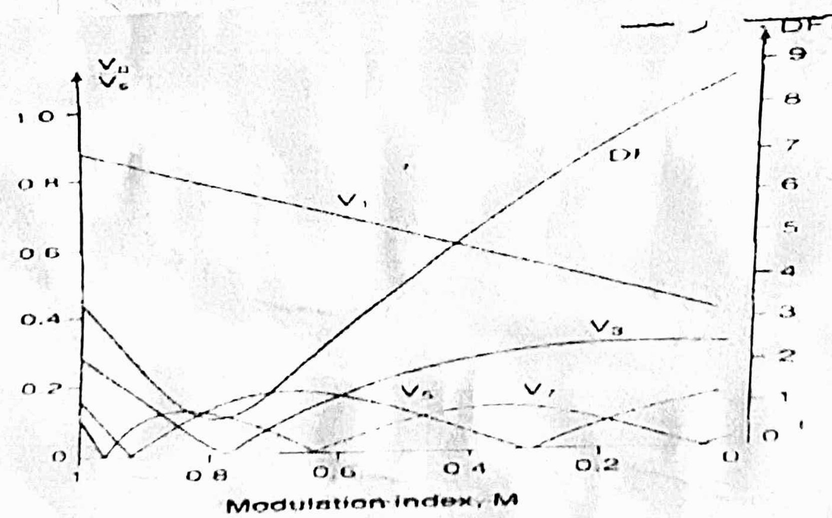


Fig 1.8 Harmonic profile

ultiple Pulse width Modulation :-

In multiple pwm control, instead of having a single pulse per half cycle, there will be multiple number of pulses per half cycle all of them being of equal width.

$f_0 = f_r$. output frequency is determined by frequency of reference signal.

f_c determines no. of pulses / half cycle.

No. of pulses / half cycle $= P = \dfrac{f_c}{2f_0} = \dfrac{m_f}{2}$.

$m_f \rightarrow$ frequency modulation ratio.

$M \rightarrow$ varied from 0 to 1

pulse width 0 to π/p.

voltage 0 to V_s.

output RMS voltage V_{or} $= \left[\dfrac{1}{\pi/p} \displaystyle\int_{(\frac{\pi}{p} - \delta)/2}^{(\pi/p + \delta)/2} V_s^2 \, d\omega t \right]^{1/2}$.

$$= V_s \sqrt{\frac{P\delta}{\pi}}.$$

Instantaneous output voltage,

Half wave symmetry $\Rightarrow a_0 = a_n = 0$.

$$b_n = \frac{V_s}{\pi}\left[\int_{\alpha_m}^{\alpha_m + \delta} \cos n\omega t \, d\omega t - \int_{\pi + \alpha_m}^{\pi + \alpha_m + \delta} \cos n\omega t \, d\omega t \right].$$

$$= \frac{V_s}{\pi}\left[\left(\frac{\sin n\omega t}{n}\right)_{\alpha_m}^{\alpha_m + \delta} - \left(\frac{\sin n\omega t}{n}\right)_{\pi + \alpha_m}^{\pi + \alpha_m + \delta} \right].$$

$$= \frac{V_s}{n\pi}\Big[\sin n(\alpha m + \delta) - \sin n\alpha m - \sin n(\pi \ldots$$
$$\sin n(\pi + \ldots$$

$\cancel{\frac{V_s}{n\pi}}$

For a two pulse,

$$A_n = \frac{2}{\pi}\int_0^{\pi} V_s \cdot \sin n\omega t \cdot d\omega t$$

$$= \frac{2}{\pi}\int_{\gamma - d/2}^{\gamma + d/2} V_s \cdot \sin n\omega t \cdot d\omega t \times 2.$$

$$= \frac{4V_s}{n\pi}\left(\cos n\omega t\right)_{\gamma - d/2}^{\gamma + d/2}$$

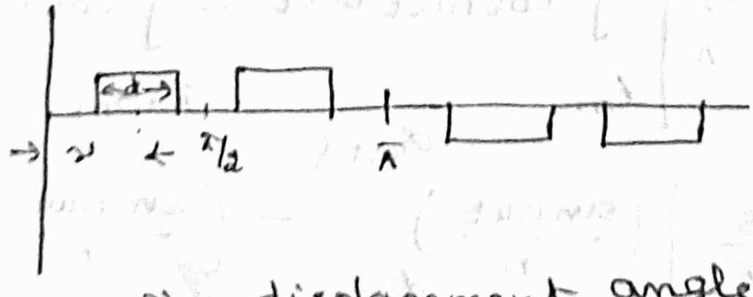
$$= \frac{4V_s}{n\pi}\Big[\cos n(\gamma - d/2) - \cos n(\gamma + d/2)\Big].$$

$$= \frac{4V_s}{n\pi}\Big[\cos n\gamma \cos \frac{nd}{2} + \sin n\gamma \sin \frac{nd}{2} - \cos n\gamma \cos \frac{nd}{2} + \sin n\gamma \sin \frac{nd}{2}\Big].$$

$$V_o = \sum \frac{8V_s}{n\pi} \sin n\gamma \sin nd/2 \; \sin n\omega t. \quad (n = 1, 3, 5 \ldots$$

$\cancel{b_n = \frac{4}{\pi}}$

$\gamma = \pi/n$, $d = \frac{2\pi}{n}$, nth harmonic eliminated.

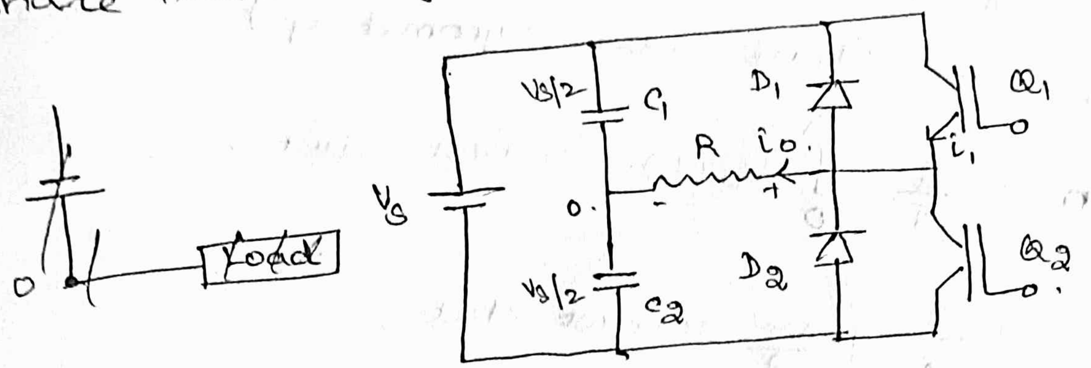


γ – displacement angle.

UNIT-IV

Inverters

ngle phase Half Bridge voltage source Inverters :-



⇒ The inverter circuit consists of two choppers. when transistor Q_1 is turned on, for a time $T_o/2$, instantaneous voltage across the load is $V_o = V_S/2$.

⇒ If transistor Q_2 is turned on, for a time $T_o/2$ to T $-V_S/2$ appears across a load.

⇒ Q_1, Q_2 are not turned on at the same time.

rms output voltage can be found from

$$V_o = \left[\frac{2}{T_0}\int_0^{T_0/2}\frac{V_S^2}{4}\,dt\right]^{1/2}, \quad V_{o(avg)} = \frac{1}{T}\int_0^T V_o\,d\omega t$$

$$= \frac{1}{\frac{T_0}{2}}\int_0^{T_0/2}\frac{V_S}{2}\,d\omega t$$

$$\frac{2}{T_0}\frac{V_S}{2}\left[T_0/2\right] = V_S/2.$$

$$= V_S/2.$$

Instantaneous output voltage can be expressed as, in fourier series

$$V_o(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n(\cos n\omega t) + b_n \sin(n\omega t)] \quad \text{---(1)}$$

$$= \frac{2}{\pi}\int_0^{\pi} V_o(\omega t)\,d\omega t.$$

$$a_0 = \frac{2}{T}\int_0^T V_o(t)\,dt$$

(half wave symmetry).

$$a_0 = 0$$

$$a_n = \frac{2}{T}\int_0^T V_o(t) . \cos n\omega t . d\omega t$$

$$= \frac{2}{\pi}\int_0^\pi V_o(t) . \cos n\omega t . d\omega t$$

$a_n = 0$ (half wave symmetry).

$$b_n = \frac{2}{T}\int_0^T V_o(t) . \sin n\omega t . d\omega t$$

$$= \frac{2}{\pi}\int_0^\pi \frac{V_s}{2} \sin n\omega t . d\omega t$$

$$= \frac{V_s}{\pi}\left[-\frac{\cos n\omega t}{n}\right]_0^\pi$$

$$= \frac{V_s}{n\pi}\left[-\cos n\pi + \cos 0\right]$$

$$= \frac{V_s}{n\pi}\left[\cos 0 - \cos n\pi\right]$$

$\cos 0 = 1$.
$\cos \pi = -1$.

when $n = 1$, $= \frac{V_s}{\pi}\left[\cos 0 - \cos\pi\right] = \frac{2V_s}{n\pi}$

$n = 2$, $= 0$.

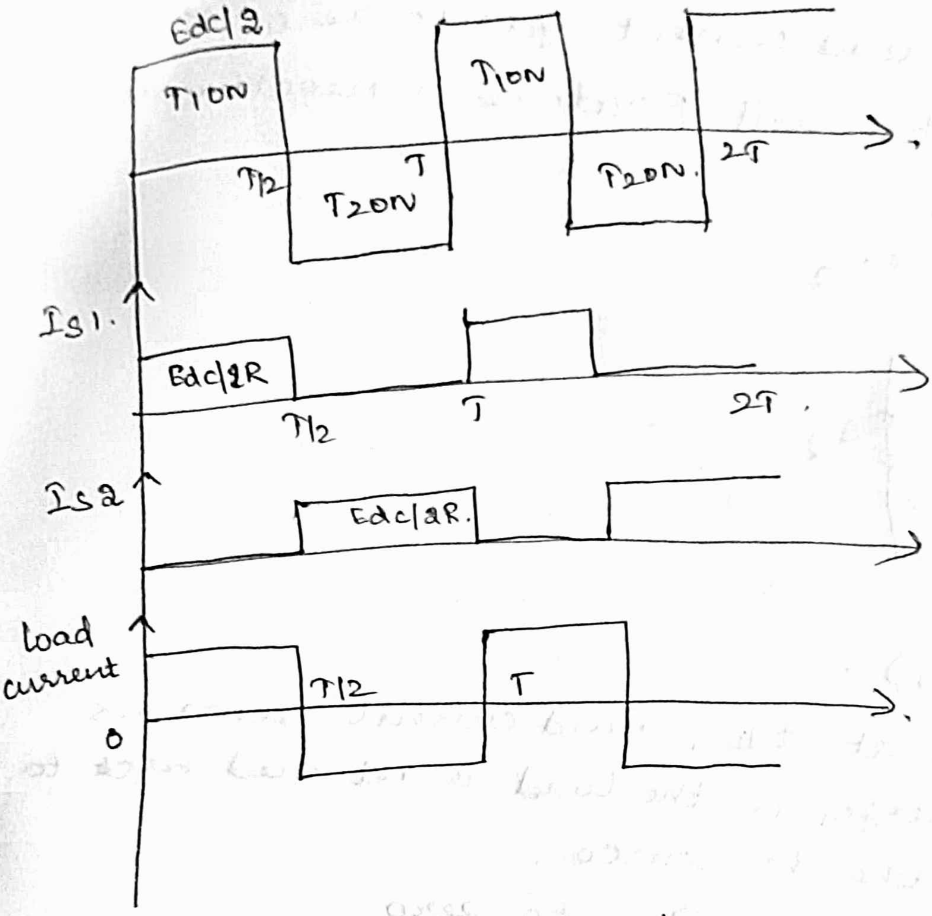
$n = 3$, $= \frac{2V_s}{n\pi}$. Sub in eqn (1).

$$\therefore V_o(t) = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$$

$$I_{T\,avg} = \frac{1}{T}\int_0^{T/2} \frac{E_{dc}}{2R}\, dt = \frac{E_{dc}}{4R}$$

$$I_{T\,rms} = \frac{E_{dc}}{2\sqrt{2}\, R}$$

$$I_{T\,peak} = \frac{E_{dc}}{2R}$$



operation with RL Load :-

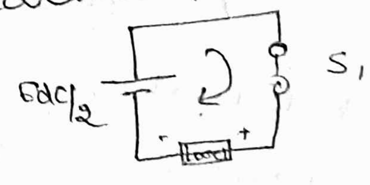
=> with an inductive load, the output voltage waveform is similar to that with a R-load, but load current cannot change immediately.

Mode 1 : ($t_1 < t < t_2$) load voltage = Edc/2.

S_1 turned on at t_1, load current reaches peak value.

At instant t_2, load current

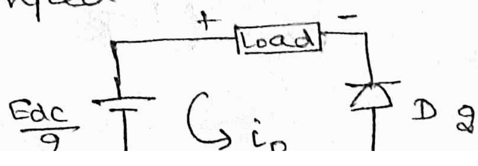
S_1 is turned off.



Mode 2 : ($t_2 < t < t_3$)

Due to inductive load, load current direction will be maintained even when S_1 is off. Stored energy in load is fed back to the lower half of the source

load voltage is clamped to $-Edc/2$.



(iii) Distortion factor (DF) :-

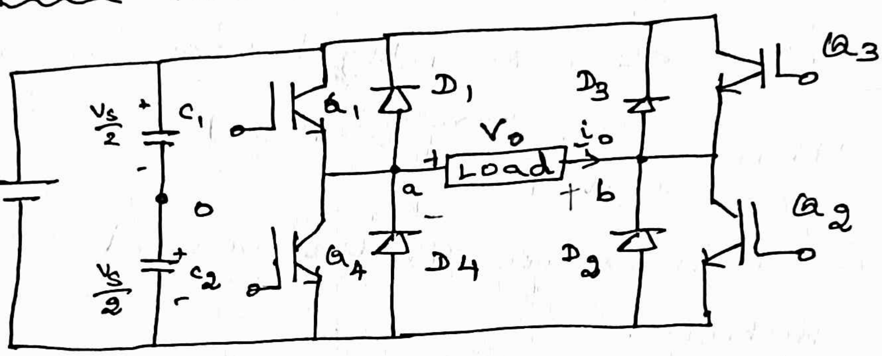
It indicates the amount of HD
remains in a particular waveform afte
harmonics of that waveform have been su
a second order attenuation.

$$DF = \frac{1}{V_{01}}\left[\sum_{n=2,3}^{\infty}\left(\frac{V_{on}}{n^2}\right)^2\right]^{1/2}.$$

iv) Lowest order harmonic (LOH) :

The LOH is that harmonic component wh
frequency is closest to the fundamental one.
Its amplitude is greater than or equal to 3% of
the fundamental component.

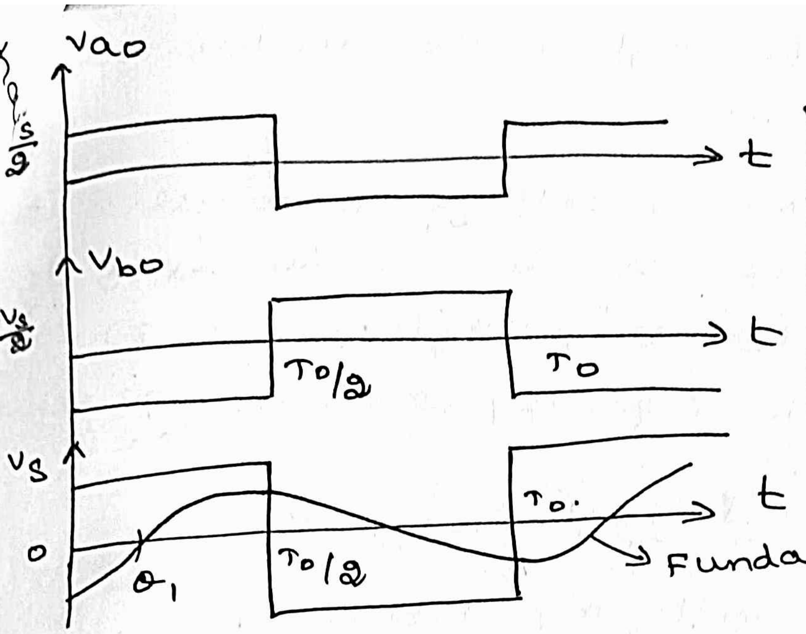
Single phase Bridge Inverters :-



It consists of four choppers. when Transistors Q_1 & Q_2 are turned ON simultaneously, the input voltage V_s appears across the load.

If transistors Q_3 & Q_4 are turned ON, the voltage across the load is reversed., and is $-V_s$.

$-V_s + V_o = 0.$



$V_o = V_a - V_b$.

switch state	V_a	V_b	$V_o = V_a - V_b$
$Q_1 Q_2$ ON	$\frac{V_s}{2}$	$-\frac{V_s}{2}$	$V_o = V_s$.
$Q_4 Q_3$ ON	$-\frac{V_s}{2}$	$\frac{V_s}{2}$	$V_o = -V_s$
$Q_1 Q_3$ ON	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$V_o = 0$.
$Q_2 Q_4$ ON	$-\frac{V_s}{2}$	$-V_s/2$	$V_o = 0$.

The rms output voltage can be found from,

$$V_o = \left(\frac{2}{T_o}\int_0^{T_o/2} V_s^2\, dt\right)^{1/2} = V_s.$$

Instantaneous output voltage in a fourier series,

$$V_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi}\sin n\omega t.$$

$a_0 = a_n = 0$.

$b_n = \frac{2}{\pi}\int_0^{\pi} V_o(t).\sin n\omega t\, d\omega t.$

$= \frac{2}{\pi}\int_0^{\pi} V_s . \sin n\omega t\, d\omega t$

$= \frac{2V_s}{n\pi}\left[-\frac{\cos n\omega t}{n}\right]_0^{\pi}$

$= \frac{2V_s}{n\pi}\left[\cos 0 - \cos\pi\right]$.

$= \frac{4V_s}{n\pi}$

for $n = 1$,

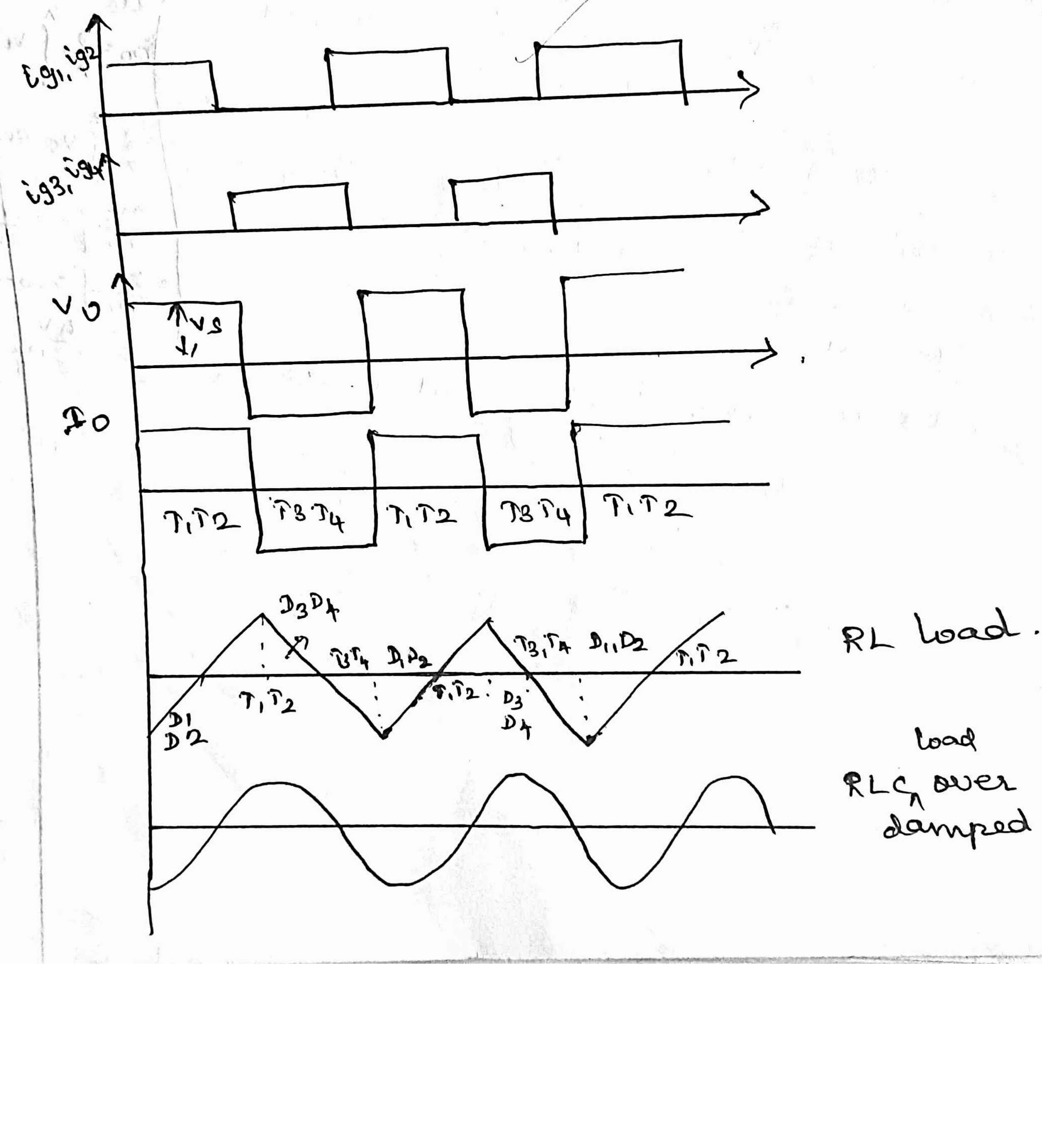
$$V_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{\sqrt{2}\pi} = 0.90 V_s.$$

rms value of fundamental component

Instantaneous load current i_o for an RL load,

$$i_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n).$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right);$$

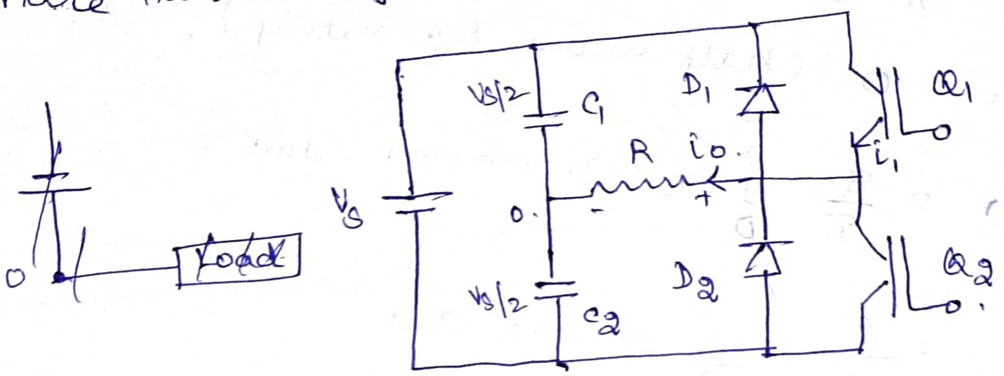


ig1, ig2
ig3, ig4
Vo
Vs
Io
T1T2
T3T4
T1T2
T3T4
T1T2
D3D4
T3T4
D1D2
T1T2
D1
D2
T1T2
D3
D4
T3,T4
D1,D2
T1T2
RL Load.
load
RLC over
damped.

UNIT-IV

Inverters.

Single phase Half Bridge voltage source Inverters :-



The inverter circuit consists of two choppers.

⇒ The inverter circuit consists of two choppers, when transistor Q_1 is turned on, for a time $T_0/2$, instantaneous voltage across the load is $V_0 = V_s/2$.

⇒ If transistor Q_2 is turned on, for a time $T_0/2$ to T, $-V_s/2$ appears across a load.

⇒ Q_1, Q_2 are not turned on at the same time.

rms output voltage can be found from

$$V_0 = \left[\frac{2}{T_0}\int_0^{T_0/2}\frac{V_s^2}{4}\,dt\right]^{1/2}$$

$$= V_s/2.$$

Instantaneous output voltage can be expressed as, in fourier series

$$V_0 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n(\cos n\omega t) + b_n \sin(n\omega t)] \quad \text{—(1)}$$

$$a_0 = \frac{2}{T}\int_0^T V_0(t).dt = \frac{2}{\pi}\int_0^{\pi} V_0(\omega t).d\omega t.$$

$a_0 = 0$ (half wave symmetry).

$$a_n = \frac{2}{T}\int_0^{T} V_o(t).\cos n\omega t.d\omega t.$$

$$= \frac{2}{\pi}\int_0^{\pi} V_o(t).\cos n.\omega t.d\omega t$$

$$a_n = 0 \quad \text{(half wave symmetry)}.$$

$$b_n = \frac{2}{T}\int_0^{T} V_o(t).\sin n\omega t.d\omega t,$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{V_s}{2}\sin n\omega t.d\omega t.$$

$$= \frac{V_s}{\pi}\left[-\frac{\cos n\omega t}{n}\right]_0^{\pi}.$$

$$= \frac{V_s}{n\pi}\left[-\cos n\pi + \cos 0\right].$$

$$= \frac{V_s}{n\pi}\left[\cos 0 - \cos n\pi\right].$$

$\cos 0 = 1.$

$\cos\pi = -1.$

when $n=1, \quad = \frac{V_s}{\pi}\left[\cos 0 - \cos\pi\right] = \frac{2V_s}{n\pi}$

$n=2, \quad = 0.$

$n=3, \quad = \frac{2V_s}{n\pi}.$

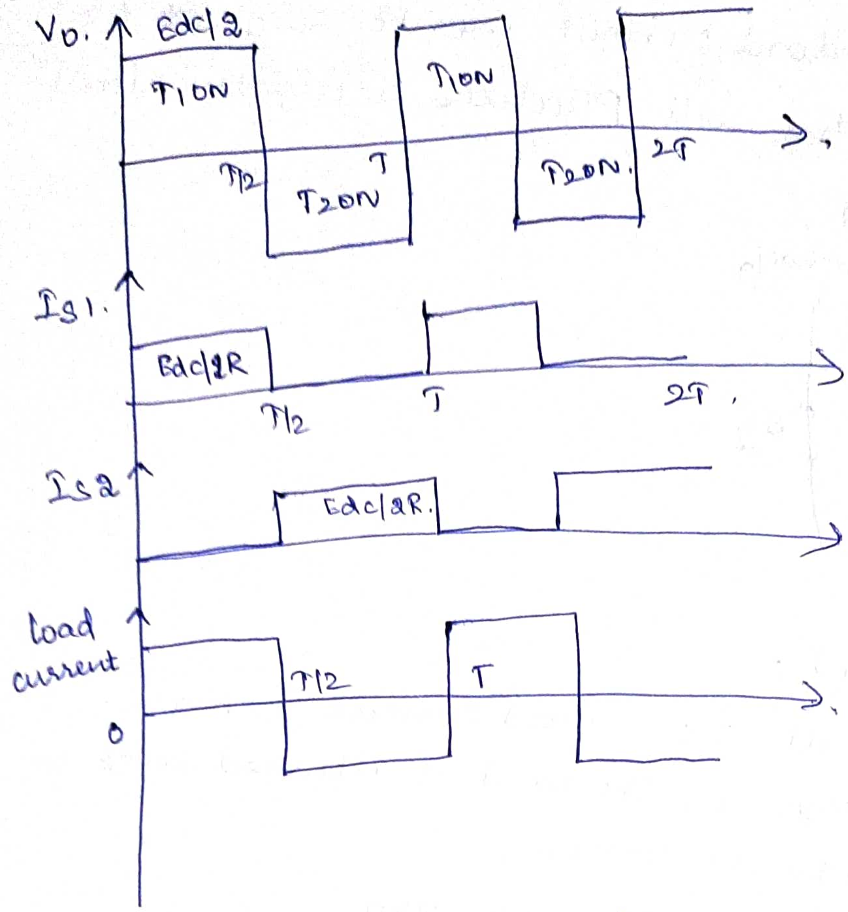
Sub in eqn (1).

$$\therefore V_o(t) = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi}\sin n\omega t.$$

$$I_{Tavg} = \frac{1}{T}\int_0^{T/2} \frac{E_{dc}}{2R}\,dt = \frac{E_{dc}}{4R}.$$

$$I_{T\,rms} = \frac{E_{dc}}{2\sqrt{2}\,R}.$$

$$I_{T\,peak} = \frac{E_{dc}}{2R}.$$

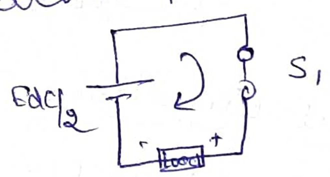


operation with RL Load :-

=) with an inductive load, the output voltage waveform is similar to that with a R-load, but load current cannot change immediately.

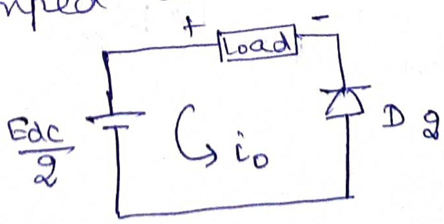
Mode 1 : ($t_1 < t < t_2$) load voltage = Edc/2.

S_1 turned on at t_1, load current reaches peak value at instant t_2, S_1 is turned off.



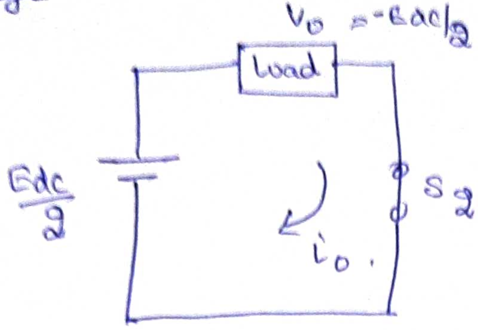
Mode 2 : ($t_2 < t < t_3$)

Due to inductive load, load current direction will be maintained even when S_1 is off. Stored energy in load is fed back to the lower half of the source load voltage is clamped to -Edc/2.



Mode (iii) ($t_3 < t < t_4$).

at instant t_3, load current goes to zero, at t_3, S_2 is turned on. This will produce a negative load voltage $e_0 = -E_{dc}/2$.

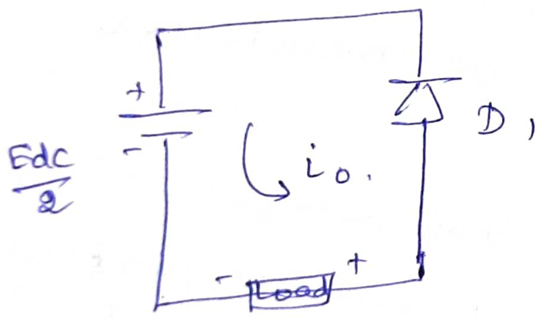


Mode 4: ($t_0 < t < t_1$):

S_2 turned off at t_4, load current remains negative, stored energy in the load is returned back to the upper half of the dc source.

at t_5, load current goes to zero,

S_1 turned ON again.



Circuit Equations:

Instantaneous current (I_0)

$$i_0(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{2E_{dc}}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n).$$

$$Z_n = \sqrt{R^2 + (n\omega L)^2}.$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right).$$

when Q_2 is turned off at $t = T_0$, load current flows through D_1, load, upper half of the d.c source.

when diode D_1 or D_2 conducts, energy is fed back to the source and these diodes are known as feedback diodes.

For an RL load, the instantaneous load current i_0 is dividing the instantaneous output voltage by the load impedance $Z = R + jn\omega L$.

$$i_0 = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n).$$

$$\theta_n = \tan^{-1} \frac{n\omega L}{R}.$$

Performance Parameters :-

(i) Harmonic factor of nth harmonic (HF_n):

$$HF_n = \frac{V_{on}}{V_{o1}} \quad \text{for } n > 1.$$

It is a measure of individual harmonic contribution.

$V_1 \rightarrow$ rms value of the fundamental component.

$V_{on} \rightarrow$ rms value of the nth harmonic component.

(ii) Total Harmonic Distortion (THD) :-

It is a measure of closeness in shape between a waveform and its fundamental component.

$$THD = \frac{1}{V_{o1}} \left[\sum_{n=2,3}^{\infty} V_{on}^2 \right]^{1/2}.$$

(iii) Distortion factor (DF) :-

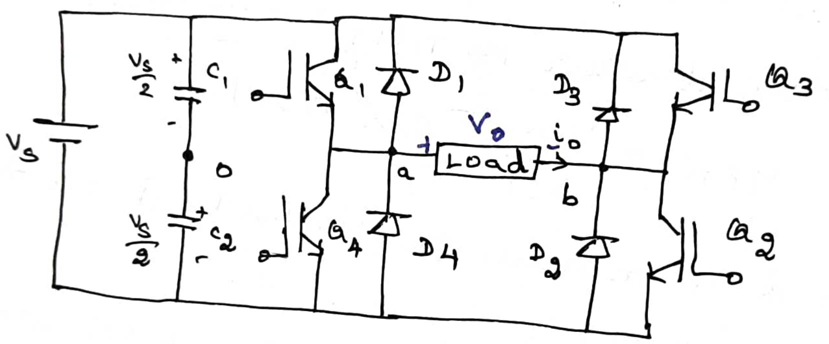
It indicates the amount of HD that remains in a particular waveform after the harmonics of that waveform have been subjected to a second order attenuation.

$$DF = \frac{1}{V_{01}}\left[\sum_{n=2,3}^{\infty}\left(\frac{V_{on}}{n^2}\right)^2\right]^{1/2}.$$

iv) Lowest order harmonic (LOH) :

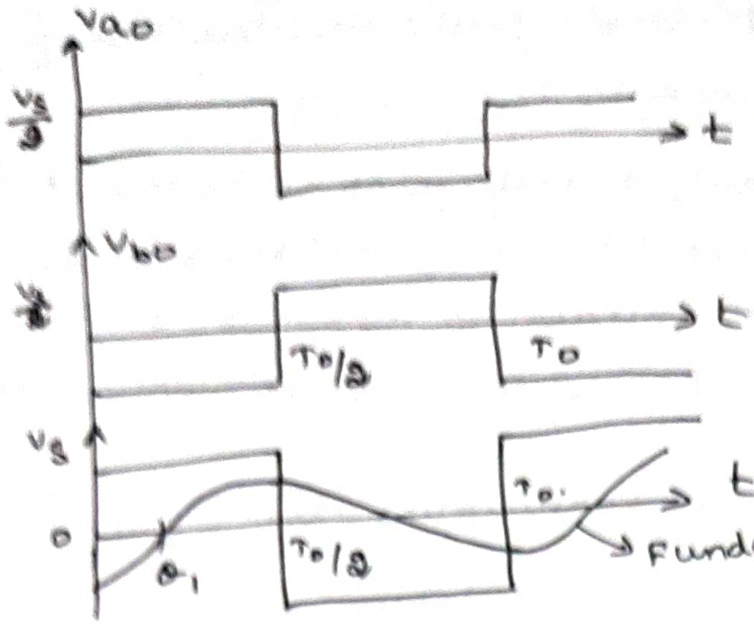
The LOH is that harmonic component whose frequency is closest to the fundamental one. Its amplitude is greater than or equal to 3% of the fundamental component.

Single phase Bridge Inverters :-



It consists of four choppers. When Transistors Q_1 & Q_2 are turned ON simultaneously, the input voltage V_s appears across the load.

If transistors Q_3 & Q_4 are turned ON, the voltage across the load is reversed, and is $-V_s$.



$V_o = V_a - V_b$.

switch state	V_a	V_b	$V_o = V_a - V_b$
$Q_1 Q_2$ ON	$\frac{V_s}{2}$	$-\frac{V_s}{2}$	$V_o = V_s$.
$Q_4 Q_3$ ON	$-\frac{V_s}{2}$	$\frac{V_s}{2}$	$V_o = -V_s$
$Q_1 Q_3$ ON	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$V_o = 0$.
$Q_2 Q_4$ ON	$-\frac{V_s}{2}$	$-V_s/2$	$V_o = 0$.

The rms output voltage can be found from,

$$V_o = \left(\frac{2}{T_0}\int_0^{T_0/2} V_s^2\, dt\right)^{1/2} = V_s.$$

Instantaneous output voltage in a fourier series,

$$v_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t.$$

$a_0 = a_n = 0$.

$b_n = \frac{2}{\pi}\int_0^{\pi} v_o(t). \sin n\omega t\, d\omega t.$

$= \frac{2}{\pi}\int_0^{\pi} V_s . \sin n\omega t\, d\omega t$

$= \frac{2V_s}{n\pi}\left[-\frac{\cos n\omega t}{n}\right]_0^{\pi}$

$= \frac{2V_s}{n\pi}\left[\cos 0 - \cos\pi\right].$

$= \frac{4V_s}{n\pi}$

for $n = 1$,

rms value of fundamental component

$$V_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{\sqrt{2}\,\pi} = 0.90 V_s.$$

Instantaneous load current i_o for an RL load.

$$i_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n).$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right).$$

(iii) Distortion chara...

voltage control of single phase inverters using various PWM techniques :-

To control the output voltage of inverters it is necessary i) to cope with the variations of dc input voltage ii) regulate voltage of inverters 3) satisfy the constant volts and frequency control requirements.

4)

The common used techniques are :

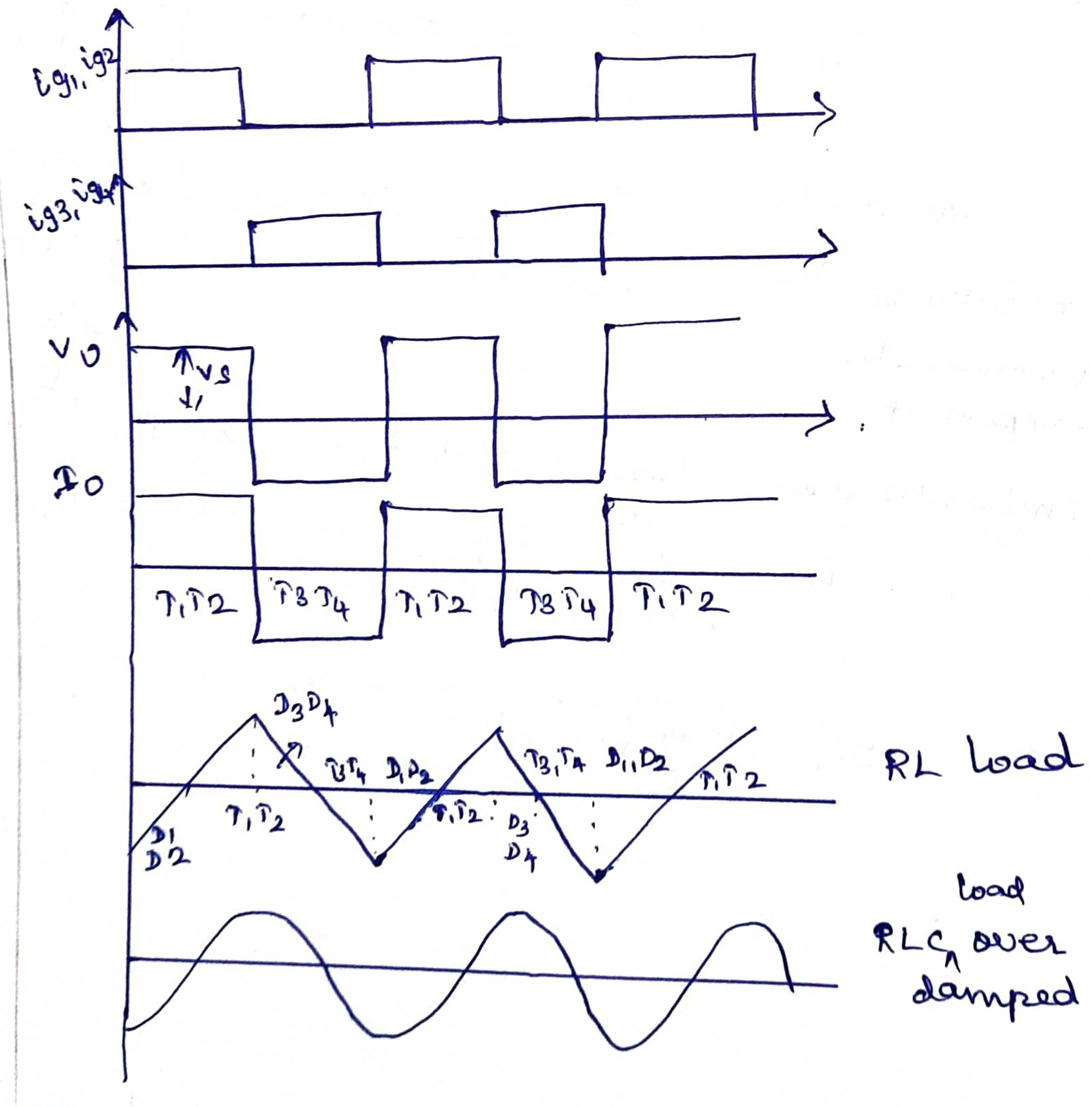
i) single pulse width modulation

ii) Multiple pulse width modulation

iii) sinusoidal pulse width modulation.

iv) Modified sinusoidal pulse width modulation

v) phase displacement control.



For RC circuit,

$$V_s = Ri_o + \frac{1}{C}\int i_o \, dt.$$

$$V_s = R \cdot \frac{dq}{dt} + q/C.$$

$V_s = R \cdot \frac{dq}{dt} + \frac{1}{C}\int \frac{dq}{dt} \, dt$

$= R \cdot \frac{dq}{dt} + q/C.$

Taking laplace Transform,

$$R\left[sQ(s) - q(0)\right] + \frac{Q(s)}{C} = \frac{V_s}{s}.$$

$\Rightarrow Q(s)$

$$Q(s) = \frac{V_s}{R} \cdot \frac{1}{s(s + 1/RC)}.$$

Taking inverse laplace Transform,

$$q(t) = CV_s\left(1 - e^{-t/RC}\right).$$

$$\boxed{V_c(t) = \frac{q(t)}{C} = V_s\left(1 - e^{-t/RC}\right).}$$

$$i_o(t) = C \cdot \frac{dv_c}{dt} = \frac{V_s}{R} \cdot e^{-t/RC}.$$

at $t = T/2$,

$$V_c(T/2) = V_s\left(1 - e^{\frac{-T}{2RC}}\right).$$

$0 \le t' \le T/2$,

$$-V_s = Ri_o + \frac{1}{C}\int i_o \, dt'.$$

$$R \cdot \frac{dq}{dt} + q/C = -V_s.$$

$$R\left[sQ(s) - CV_c(T/2)\right] + \frac{Q(s)}{C} = -\frac{V_s}{s}.$$

$$RSQ(s) - RCV_c(T/2) + \frac{Q(s)}{C} = -\frac{V_s}{s}$$

$$Q(s)\left[RS + \frac{1}{C}\right] = -\frac{V_s}{s} + RCV_c(T/2)$$

$$Q(s) = -\frac{V_s}{S}\left[\frac{C}{CRS+1}\right] + RcV_c(T/2)\frac{\times C}{RCS+1}$$

$$= \frac{-CV_s}{S(1+RCS)} + \frac{CV_c(T/2) \cdot \cancel{RC}}{(S + 1/RC) \times \cancel{RC}}$$

$$= \frac{-CV_s}{S(1+RCS)} + \frac{CV_c(T/2)}{\left(S + \frac{1}{RC}\right)}$$

$$V_c(t') = -V_s + V_s\left[2 - e^{-\frac{T}{2RC}}\right]e^{-\frac{t'}{RC}}$$

$$\boxed{i_o(t') = \frac{C \cdot dV_c(t')}{dt} = -\frac{V_s}{R}\left(2 - e^{-\frac{T}{2RC}}\right) \cdot e^{-\frac{t'}{RC}}}$$

under steady state conditions,

at $t=0$; $i_o(0) = -I_o$.

$$\frac{V_s}{S} = I(s)\left[R+LS\right] + L \cdot I_o.$$

$$i_o(t) = \frac{V_s}{R}\left(1-e^{-R/L\,t}\right) - I_o \cdot e^{-R/L\,t}.$$

at $t = T/2$, $i_o(t) = I_o$

$$i_o(T/2) = I_o = \frac{V_s}{R}\left(1-e^{-\frac{RT}{2L}}\right) - I_o e^{-\frac{RT}{2L}}.$$

$$I_o = \frac{V_s}{R}\,\frac{1-e^{-\frac{RT}{2L}}}{1+e^{-\frac{RT}{2L}}}.$$

$$\boxed{i_o(t) = \frac{V_s}{R}\left[1-e^{-\frac{R}{L}t}\right] - \frac{V_s}{R}\,\frac{1-e^{-\frac{RT}{2L}}}{1+e^{-\frac{RT}{2L}}}\,e^{-\frac{R}{L}t}}$$

at $t = T/2$; $i_o(T/2) = I_o$.

$$\frac{-V_s}{R} = I(s)\left[R+LS\right] - LI_o.$$

$$= -\frac{V_s}{R}\left[1-e^{-R/L\,t'}\right] + I_o e^{-\frac{R}{L}t'}.$$

$$I(s) = \frac{-V_s}{s(R+LS)} + \frac{L \cdot i_o(T/2)}{R+LS}.$$

$$i_o(t') = \frac{-V_s}{R}\left(1-e^{-\frac{R}{L}t'}\right) + i_o(T/2)e^{-R/L\,t}.$$

$$= -\frac{V_s}{R}\left(1-e^{-R/L\,t'}\right) + \frac{V_s}{R}\left(1-e^{-\frac{RT}{2L}}\right).$$

$$= -\frac{V_s}{R} + \frac{V_s}{R}\left(2e^{-R/L\,t'}\right) + \frac{V_s}{R}e^{-\frac{RT}{2L}} \cdot e^{-R/L\,t'}.$$

$$\boxed{i_o(t') = -\frac{V_s}{R} + \frac{V_s}{R}\left[2 - e^{-\frac{RT}{2L}}\right] e^{-\frac{R}{L}t'}}$$

$0 \leq t' \leq T/2.$

under steady state conditions,

at $t=0$; $i_o(0) = -I_o$.

$$\frac{V_s}{s} = I(s)\left[R + Ls\right] + LI_o.$$

$$i_o(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right) - I_o \cdot e^{-R/L\, t}.$$

at $t = T/2$, $i_o(t) = I_o$.

$$i_o(T/2) = I_o = \frac{V_s}{R}\left(1 - e^{-\frac{RT}{2L}}\right) - I_o \cdot e^{-\frac{RT}{2L}}.$$

$$I_o = \frac{V_s}{R} \cdot \frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}}.$$

$$\boxed{i_o(t) = \frac{V_s}{R}\left[1 - e^{-\frac{R}{L}t}\right] - \frac{V_s}{R}\,\frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}} \cdot e^{-\frac{R}{L}t}}$$

at $t = T/2$, $i_o(T/2) = I_o$.

$$-\frac{V_s}{R} = I(s)\left[R + Ls\right] - LI_o.$$

$$= -\frac{V_s}{R}\left[1 - e^{-\frac{R}{L}t'}\right] + \frac{V_s}{R}\,\frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}}\, e^{-\frac{R}{L}t'}.$$

If $T/2 - t_1 > t_q \rightarrow$ Natural commutation.

RL load :-

$$V_s = Ri_o + L\frac{di_o}{dt} \quad 0 \le t \le T/2.$$

L.T,

$$\frac{V_s}{s} = RI(s) + L[sI(s) - \underbrace{I(0)}_{\to 0}] = RI(s) + LsI(s)$$

$$= I(s)[R + Ls].$$

$$I(s) = \frac{V_s}{s(R+Ls)} = \frac{V_s}{sL\left(\frac{R}{L}+s\right)} \qquad \frac{V_s}{sL[R/L+s]}$$

$$\frac{V_s}{s(R+Ls)} = \frac{A}{s} + \frac{B}{R+sL}.$$

$V_s = A(R+sL) + Bs.$

Put $s=0$, $V_s = AR$, $\boxed{A = V_s/R}$

Put $s = -R/L$

$V_s = \frac{V_s}{R}[0] + B(-R/L).$

$\boxed{B = -\frac{V_s \cdot L}{R}}$

$L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$

$$\frac{A}{s} + \frac{Bs+C}{(R/L+s)}$$

$$\boxed{i(t) = \frac{V_s}{R}\left(1 - e^{-R/L\, t}\right)}.$$

at $t = T/2$,

$$i_o(T/2) = \frac{V_s}{R}\left(1 - e^{-R/L \cdot T/2}\right).$$

$T/2 \le t \le T$ $\quad -V_s' = Ri_o + L\frac{di_o}{dt} \quad T/2 \le t \le T.$

$-\frac{V_s}{s} = RI(s) + LsI(s) - Li_o(T/2)$

$$I(s) = \frac{-V_s}{s(R+Ls)} + \frac{L \cdot i_o(T/2)}{R+Ls}.$$

$$i_o(t') = \frac{-V_s}{R}\left(1 - e^{-R/L\, t'}\right) + i_o(T/2)e^{-R/L\, t}.$$

$$\boxed{i_o(t') = \frac{-V_s}{R} + \frac{V_s}{R}\left[2 - e^{-\frac{RT}{2L}}\right]e^{-R/L\, t'}}$$

$$\frac{V_s}{Ls[s+R/L]} = \frac{A}{s} + \frac{Bs+C}{s+R/L}$$

$A(s+R/L) + Bs + C(s)$

R/L

$-V_s = Ri_o + L\frac{di_o}{dt}$; $T/2 \le t \le T.$

$-\frac{V_s}{s} = RI(s) + L[sI(s) - I(0)].$

$-V_s/s = RI(s) + L\,sI(s) - L\,I(0)$

$-V_s/s = RI(s) + L\,sI(s) - L\,I(0).$

$-V_s/s = I(s)[R + sL] - L\,I(0).$

$-V_s/s = I(s)(R+sL).$

$-V_s/s + L\,I(0) = I(s)(R+sL)$

$I(s) = \frac{-V_s}{s(R+sL)} + \frac{L \cdot I(0)}{R+sL}$

$0 \leq t \leq T/2$

$$V_s = Ri_o + L\frac{di_o}{dt} + \frac{1}{C}\int i_o dt + V_{C1}.$$

$V_{C1} \rightarrow$ voltage across the capacitor at $t=0$.

$T/2 \leq t \leq T$ (or) $0 \leq t' \leq T/2$, $t' = t - T/2$.

$$-V_s = Ri_o + L\frac{di_o}{dt} + \frac{1}{C}\int i_o dt' + V_{C2}.$$

$V_{C2} \rightarrow$ voltage across capacitor at $t'=0$.

Differentiating both the equations,

$$\frac{d^2 i_o}{dt^2} + \frac{R}{L}\frac{di_o}{dt} + \frac{1}{LC} i_o = 0.$$

$$\frac{d^2 i_o}{dt'^2} + \frac{R}{L}\frac{di_o}{dt'} + \frac{1}{LC} i_o = 0.$$

solving these two equations, i_o will be obtained

In RL and RLC over damped:

At $t=0$, T_1, T_2 are triggered. But the current direction cannot be change immediately, $D_1, D_2 \rightarrow$ starts conduct.

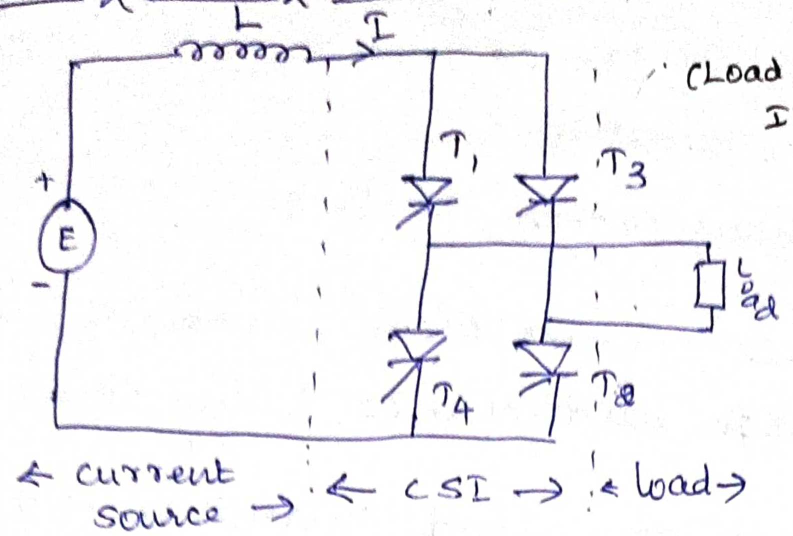
at $t = T/2$, $T_1; T_2$ are force commutated.

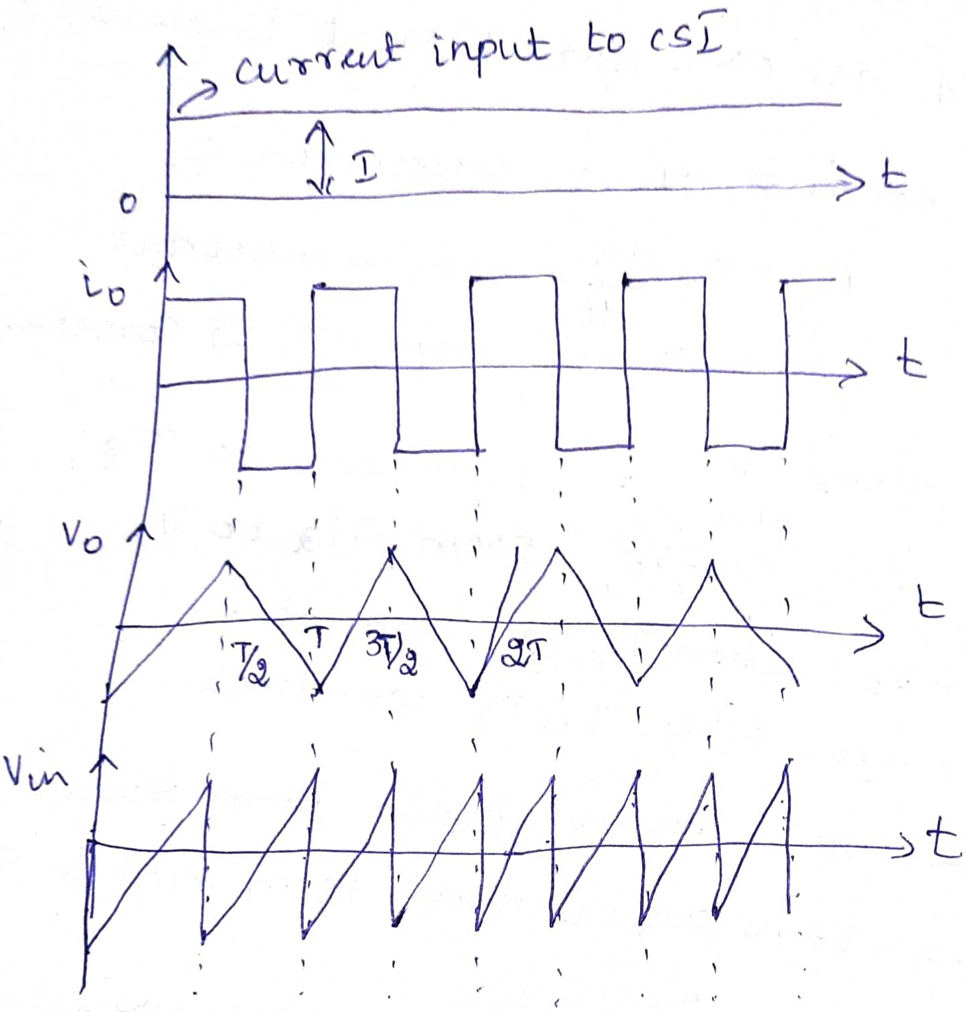
RLC underdamped load :-

after $t=0$, T_1, T_2 conduct. But due to the load nature, at $t=t_1$, T_1, T_2 are ~~turned~~ off, D_1, D_2 into action.

Current Source Inverters :-

UNIT - IV





⇒ Current source inverter (CSI), input current is constant but adjustable current.

⇒ The amplitude of current from CSI is independent of the load.

⇒ Does not require any feedback diodes.

applications :-

i) speed control of ac motors

ii) Induction heating

iii) synchronous motor starting.

⇒ The source consists of a voltage source E and a large inductance L in series with it.

⇒ T_1, T_2 are ON, load current $i_o \rightarrow$ +ve. $i_o = I$.

T_3, T_4 are ON, load current $i_o \rightarrow$ -ve, $i_o = -I$.

⇒ Load consists of a capacitor C,

$$i_o = C \cdot \frac{dv_o}{dt}. \qquad i_o \rightarrow \text{constant}$$

$$\text{slope } \frac{dv_o}{dt} \rightarrow \text{constant}.$$

⇒ This slope is +ve, from zero to $T/2$,

-ve from $T/2$ to T.

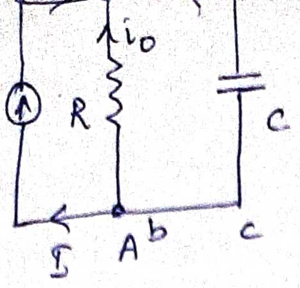
$V_{in} = V_o$, when T_1, T_2 conduct.

$V_{in} = -V_o$, when T_3, T_4 conduct.

⇒ $V_{in} \rightarrow$ +ve, power flows from source to load.

$V_{in} \rightarrow$ -ve, power flows from load to source.

⇒ CSI may be load or force commutated.



$$i_o + i_c + I = 0.$$

$$i_c = -I - i_o.$$

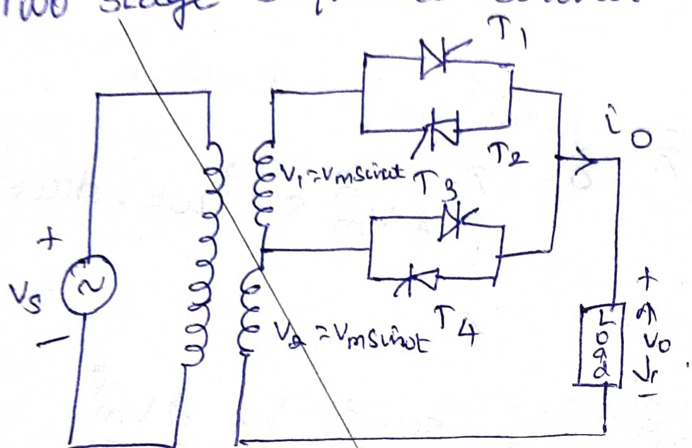
At $t = T/2$, $i_o = +I_1$

$$i_c = -I - I_1 = -(I + I_1).$$

At $t = T$; $i_o = -I_1$,

$$i_c = -I + I_1 = -(I - I_1).$$

Two stage sequence control of voltage controllers :-



$$V_s = V_m \sin\omega t$$

$$V_1 = V_2 = V_m \sin\omega t.$$

Sum of two secondary voltages is $2V_m \sin\omega t$.

Advantage :-

Reduction of harmonics in the load and supply currents.

Resistance Load :

when both pairs T_1, T_2 & T_3, T_4 are in operation, firing angle for for T_3, T_4 is always zero, & for pair T_1, T_2 is varied from 180° to zero, for obtaining output voltage from V to 2V.

⇒ SCR T_1 is triggered, at $\omega t = \alpha$,
V_1 reverse biases T_3, it is turned off.

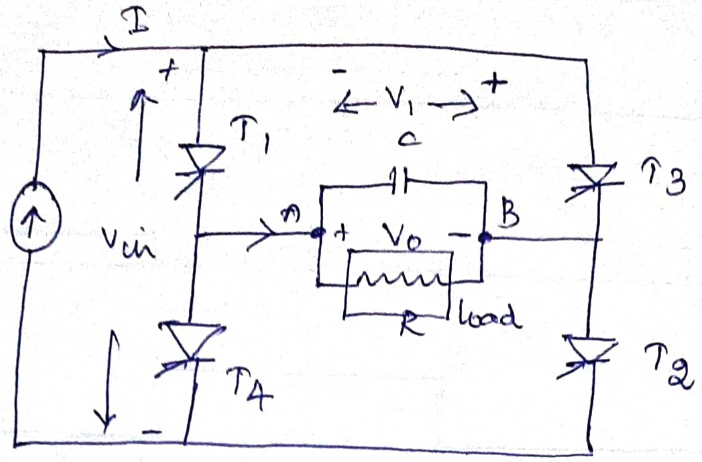
⇒ T_1 begins to conduction, output voltage jumps from V_2 to $(V_1 + V_2)$.

⇒ T_4 is triggered → output voltage follows $V_m \sin \alpha$

$$V_{or} = \left[\frac{1}{\pi}\int_0^{\alpha} V_m^2 \sin^2 \omega t \, d(\omega t) + \int_{\alpha}^{\pi} 4V_m^2 \sin^2 \omega t \, . d(\omega t)\right]^{1/2}$$

$$= \frac{V_m^2}{2\pi}\left(\alpha - \frac{\sin 2\alpha}{2}\right) + \frac{2V_m^2}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)^{1/2}.$$

1 ϕ capacitor - commutated CSI with R-load :-



→ capacitor C in parallel with the load is used for storing the charge for force commutating the SCRs.

→ T_1, T_2 together gated by i_{g1}, i_{g2}.
T_3, T_4 together gated by i_{g3}, i_{g4}.

Before $t=0$, $V_c = -V_1$, left plate -ve,
Right plate +ve.

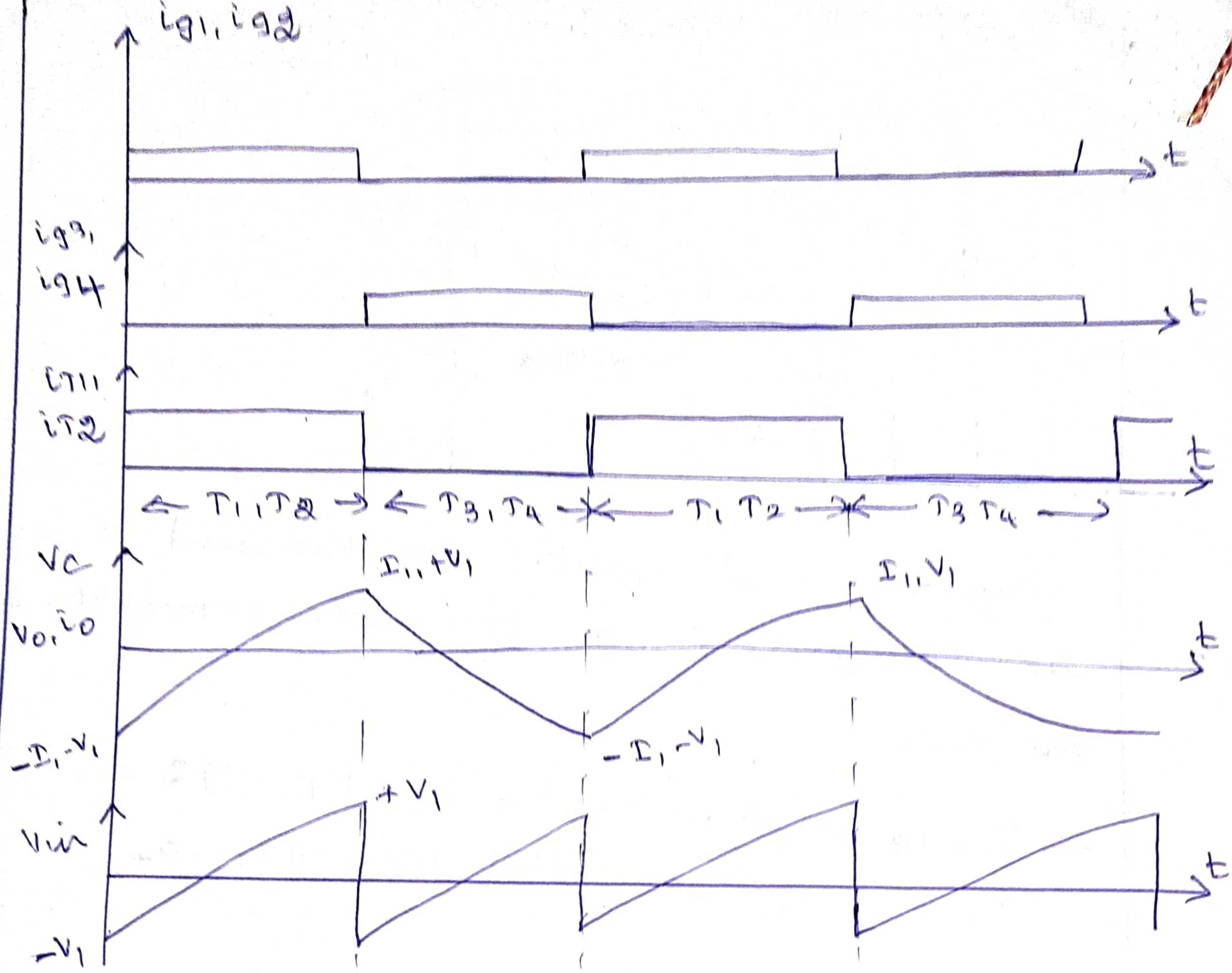
→ when T_1, T_2 are gated at $t=0$, V_c reverse biases conducting thyristors T_3, T_4.

→ source current I flows through T_1, parallel combination of R and C through T_2.

→ From zero to $T/2$, $i_{T1} = i_{T2} = I$.

→ when T_3, T_4 are gated at $t = T/2$, $V_c = V_1$
T_1, T_2 reverse biases.

→ Source current I flows through T_3, Parallel combination of R and C, $T/2$ to T.

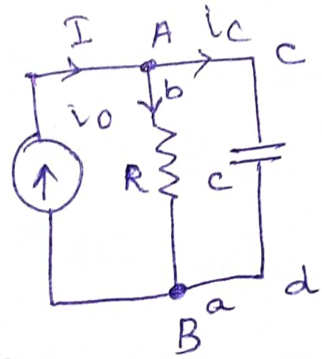


At $t = 0$, capacitor charged with $V_c = -V_1$.

load current $i_0 = \dfrac{-V_1}{R_1} = -I_1$.

$t = 0$ to $T/2$; capacitor charges from $-V_1$ to V_1.

$t = T/2$; $i_0 = \dfrac{V_0}{R} = \dfrac{V_1}{R} = I_1$.



KCL at node A,

$$i_0 + i_c = I$$

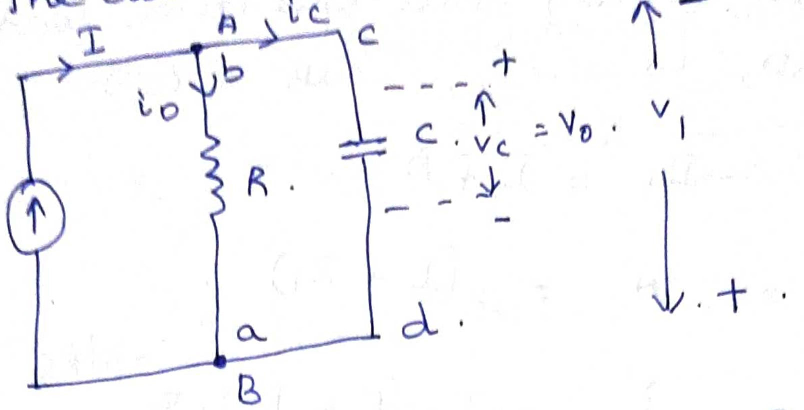
$$i_c = I - i_0.$$

At $t = 0$, $i_0 = -I_1$ $i_c = I + I_1$.

before $T/2$, $i_0 = I_1$, $i_c = I - I_1$.

Analysis :-

The capacitor is initially charged to a voltage $-V_1$. The closed path abcda we get,



$$Ri_0 - \frac{1}{C}\int (I - i_0)\,dt + V_1 = 0. \quad \text{--- (1)}$$

differentiate with respect to time,

$$R.\frac{di_0}{dt} - \frac{I}{C} + \frac{i_0}{C} = 0.$$

$$R\frac{di_0}{dt} + \frac{i_0}{C} = \frac{I}{C}.$$

$$\left[Rp + \frac{1}{C}\right] i_0 = \frac{I}{C}. \quad \text{--- (2)}.$$

complementary ~~solution~~ function of the solution is $\left[Rp + \frac{1}{C}\right] I_{cp} = 0.$

$$Rp = -\frac{1}{C}.$$

$$\boxed{p = -\frac{1}{RC}}.$$

$$\boxed{I_{cp} = A.e^{-t/RC}}$$

For particular integral, put $p = 0$,

$$\frac{i_0}{C} = \frac{I}{C} \qquad (\text{or } i_0 = I.$$

complete solution for load current i_o,

$$i_o = P.I + C.F.$$

$$i_o = I + A.e^{-t/RC} \quad \text{--- (3).}$$

under steady state operation, load current at $t=0$, $i_o = -I_1$, sub in (3).

$$-I_1 = I + A.$$

$$A = -(I + I_1).$$

$$i_o = I - (I + I_1).e^{-t/RC}.$$

$$i_o = I - I.e^{-t/RC} - I_1.e^{-t/RC}.$$

$$i_o = I\left[1 - e^{-t/RC}\right] - I_1.e^{-t/RC}.$$

$$0 < t < T/2. \quad \text{--- (4).}$$

at $t = T/2$, $i_o = I_1$. sub in (4),

$$I_1 = I\left[1 - e^{-\frac{T}{2RC}}\right] - I_1.e^{-\frac{T}{2RC}}.$$

$$I_1 + I_1 e^{-\frac{T}{2RC}} = I\left[1 - e^{-\frac{T}{2RC}}\right].$$

$$I_1\left[1 + e^{-T/2RC}\right] = I\left[1 - e^{-\frac{T}{2RC}}\right]. \quad \text{--- (5).}$$

$$\boxed{I_1 = I\left[\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right]}$$

$$= I \quad \text{if } \frac{T}{2RC} >> 1,$$

$$T >> RC.$$

sub eqn ⑤ in eqn ④,

$$i_o = I\left[1-e^{-t/RC}\right] - I\left[\frac{1-e^{\frac{-T}{2RC}}}{1+e^{-T/2RC}}\right]e^{-t/RC}.$$

$$i_o = \frac{I\left[1-e^{-t/RC}\right]\left[1+e^{-T/2RC}\right] - I\left[1-e^{-T/2RC}\right]e^{-t/RC}}{1+e^{-T/2RC}}.$$

output voltage V_o (or) capacitor voltage V_c is,

$$V_o = V_c = Ri_o = RI\left[1-2\frac{e^{-t/RC}}{1+e^{(-T/2RC)}}\right].$$

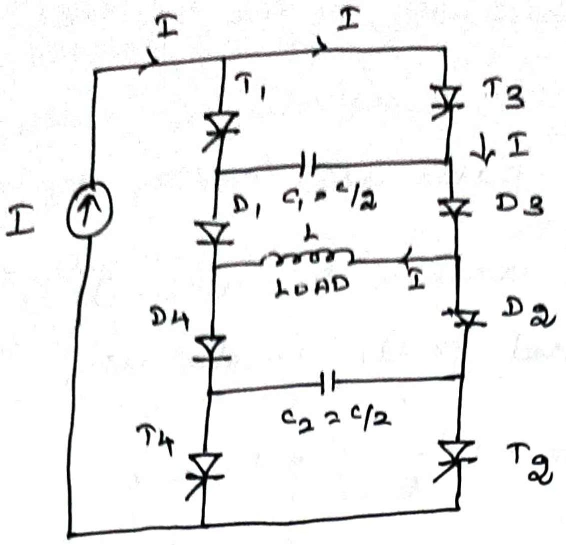
$$i_o = I\left[1-2\frac{e^{-t/RC}}{1+e^{(-T/2RC)}}\right].$$

turn ~~off~~ time t_c, provided by the circuit to each SCR is obtained when $t = t_c$, $V_o = V_c = i_oR = 0$.

$$V_o = V_c = Ri_o = RI\left[1-\frac{2e^{-t_c/RC}}{1+e^{-T/2RC}}\right] = 0.$$

$$\boxed{t_c = RC\ln\left[\frac{2}{1+\exp(-T/2RC)}\right]}.$$

Single phase Auto - sequential Commutated Inverter :-

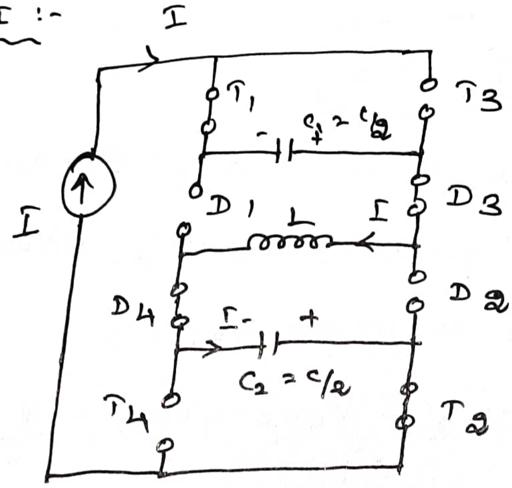


=> Thyristor pairs T_1, T_2, T_3, T_4 are alternatively switched to obtain a nearly square wave load current.

=> Two commutating capacitors one C_1 in the upper half and the other C_2 in the lower half are connected.

=> Diodes D_1 to D_4 are connected in series to prevent the commutation capacitors from discharging into the load.

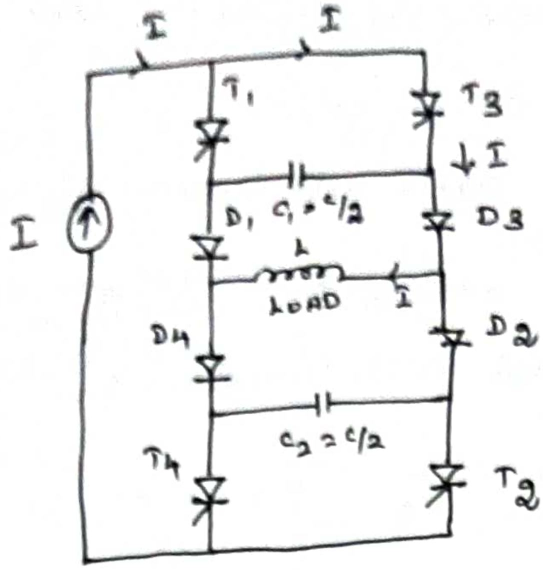
MODE I :-



before $t = 0$, assume that T_3, T_4 are conducting and a steady current I flows through the path T_3, D_3, L, D_4, T_4 and source I.

=> commutating capacitors are assumed to be initially charged equally with polarity $V_{C1} = V_{C2} = -V_{C0}$.

Single phase Auto - sequential Commutated Inverter :-



=> Thyristor pairs T_1, T_2, T_3, T_4 are alternatively switched to obtain a nearly square wave load current.

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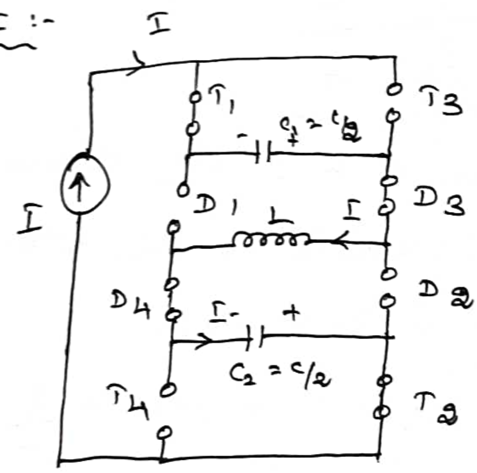
MODE I :-



before $t = 0$, assume that T_3, T_4 are conducting and a steady current I flows through the path T_3, D_3, L, D_4, T_4 and source I.

=> commutating capacitors are assumed to be initially charged equally with polarity $V_{C1} = V_{C2} = -V_{C0}$.

At $t = 0$, T_1, T_2 are gated.

T_3, T_4 are turned off by the reverse capacitor voltages.

T_1, T_2 conducts, path are T_1, C_1, D_3, L, D_4, C_2, T_2.

The voltage V_{D1} across D_1, when it is forward biased, by closed path abcda as,

$$V_{D1} + V_{co} - \frac{1}{C/2} \int I\,dt = 0.$$

Voltage across L is zero, because of constant current I.

$$V_{D1} = -V_{co} + \frac{2}{C} \int I\,dt.$$

capacitor charges, Voltages V_{D1} across D_1 rises linearly.

at $t = t_1$, $V_{D1} = 0$,

$$0 = -V_{co} + \frac{2}{C} \int I\,t_1.$$

$$\boxed{t_1 = \frac{C}{2I} V_{co}}$$

$V_{C1} = V_{C2} = V_c$ appears as reverse capacitor voltage when T_1, T_2 gated. voltage across thyristors T_3, T_4.

The value of V_c is given as,

$$V_{C1} = V_{C2} = V_c = -V_{co} + \frac{2}{C} \int I\,dt.$$

at time t_1 $\quad V_{C1} = V_{C2} = V_c(t_1) = -V_{co} + \frac{2}{C} I\,t_1.$

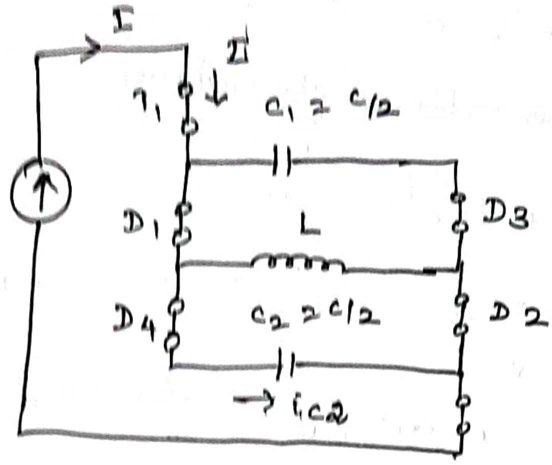
sub $t_1 = \frac{C}{2I} V_{co}$.

$$V_{C1} = V_{C2} = V_c(t_1) = -V_{co} + \frac{2I}{C}\left(\frac{C}{2I} V_{co}\right) = 0.$$

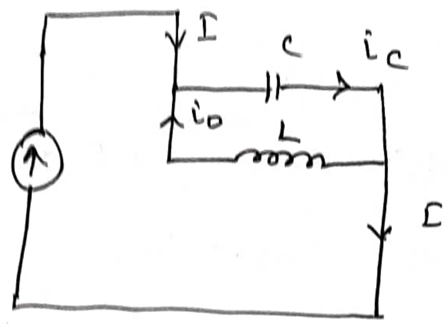
MODE II :-

Diodes D_2, D_4 are already conducting but $t = t_1$, diodes D_1, D_2 get forward biased and start conducting.

2) at the end of t_1, all four diodes D_1, D_2, D_3, D_4 conduct.



MODE II.



Equivalent circuit

In equivalent circuit, KCL gives

$$I + i_o = i_c \quad (= i_{c1} + i_{c2}).$$

$$i_{c1} = i_{c2}, \quad i_{c1} = i_{c2} = \frac{i_c}{2}.$$

KVL gives $L \cdot \frac{di_o}{dt} + \frac{1}{C}\int i_c \cdot dt = 0.$

$$L \cdot \frac{di_o}{dt} + \frac{1}{C}\int (I + i_o)\,dt = 0.$$

$$L \cdot \frac{d^2 i_o}{dt^2} + \frac{i_o}{C} = -\frac{I}{C} \quad \ldots\ldots \text{(I)}.$$

$$\frac{d^2 i_o}{dt^2} + \frac{i_o}{CL} = -\frac{I}{LC}.$$

solving the equation,

$$i_o = I, \quad \frac{di_o}{dt} = 0.$$

In eqn (I), for particular integral,

$$\frac{i_{os}}{C} = -\frac{I}{C}.$$

$$i_{os} = -I.$$

For complementary function,

$$\left(Lp^2 + \frac{1}{C}\right) i_o = 0.$$

$$Lp^2 + \frac{1}{C} = 0.$$

$$p^2 = -\frac{1}{LC} = -\omega_o^2 = j^2 \omega_o^2.$$

$$p = \pm j\omega_o.$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$i_{ot}(t) = A \cdot e^{j\omega_o t} + B e^{-j\omega_o t}.$$

$$i_o(t) = i_{os} + i_{ot}.$$

$$= -I + A e^{j\omega_o t} + B e^{-j\omega_o t} \quad \ldots\ldots (1)$$

$$t = 0, \quad i_o = I.$$

$$I = -I + A + B.$$

$$\boxed{A + B = 2I} \quad \ldots\ldots (2).$$

at $t = 0$, $\frac{di_o}{dt} = 0$, from (1),

$$\frac{di_o}{dt} = j\omega_o A e^{j\omega_o t} - j\omega_o B e^{-j\omega_o t} = 0.$$

$$j\omega_o (A - B) = 0.$$

$$(A - B) = 0. \quad \ldots\ldots (3).$$

i_0 consists of two components :-

(i) steady state component

(ii) Transient component.

Transient component $i_0 = A\cos\omega_0 t + B\sin\omega_0 t$.

steady state component :

$$L\cdot\frac{di_0}{dt} + \frac{1}{C}\int(i_0 + I)\cdot dt = 0.$$

$$\frac{di_0}{dt} = 0.$$

$$\frac{i_0 + I}{C} = 0.$$

$$i_0 = -I.$$

Total current $i_0 = -I + A\cos\omega_0 t + B\sin\omega_0 t$.

at $t = 0$, $i_0 = I$

$$i_0 = -I + A.$$

$$A = I + I = 2I.$$

$$i_0 = -I + 2I\cos\omega_0 t.$$

$$\boxed{i_0 = I\left[2\cos\omega_0 t - 1\right]}$$

$$i_c = i_0 + I$$

$$i_c = I\left[2\cos\omega_0 t - 1\right] + I.$$

$$i_c = I\left[2\cos\omega_0 t\right].$$

Voltage across the capacitor,

$$V_C = \frac{1}{C}\int i_c\cdot dt.$$

$$= \frac{1}{C}\int I(2\cos\omega_0 t)\cdot dt.$$

$$= I/C\;\frac{2\sin\omega_0 t}{\omega_0} = \frac{2I}{\omega_0 C}\cdot\sin\omega_0 t.$$

$i_{c1} = i_{c2} = \frac{i_c}{2} = \frac{2I\cos\omega_o t}{2} =$

$i_{D3} = I - i_{c1}$.

$= I - I\cos\omega_o t$.

$= I[1 - \cos\omega_o t]$.

A time t_2 must elapse for the current i_{c1} to becomes zero. This time t_2 can be obtained by equating i_{c1} to zero.

$i_{c1} = I\cos\omega_o t_2 = 0$.

$\cos\omega_o t_2 = \cos \pi/2$.

$$\boxed{t_2 = \frac{\pi}{2\omega_o}}.$$

Total commutation interval t_c is

$t_c = t_1 + t_2 = \frac{C}{2I} V_{co} + \frac{\pi}{2\omega_o}$.

$t_1 = \frac{C}{2I} V_{co} = \frac{C}{2I} \times \frac{2I}{\omega_o C}$.

$= \frac{1}{\omega_o} = \sqrt{LC}$.

$t_c = \sqrt{LC} + \frac{\pi}{2}\sqrt{LC} = \sqrt{LC}\left[1 + \pi/2\right]$.

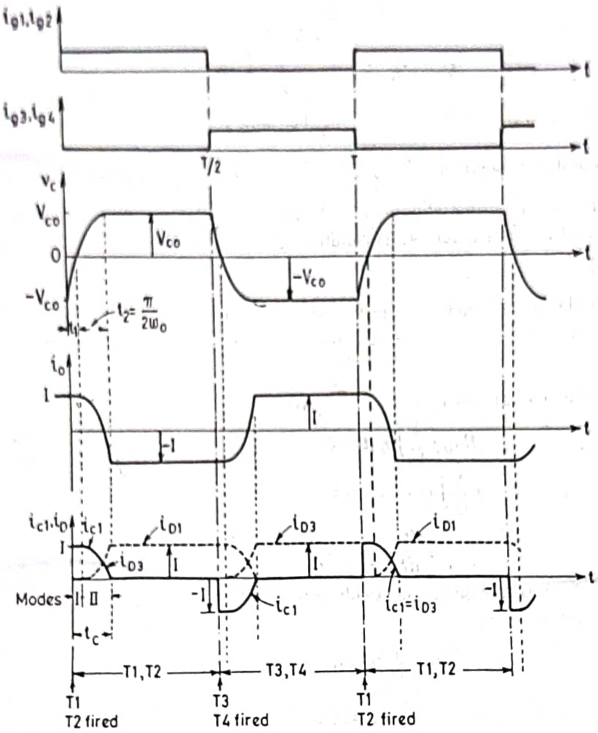
At the end of total commutation interval $(t_1 + t_2)$, the steady input current I flows through T_1, D_1, L, D_2, T_2. This current continues to flow till the next commutation process.

from eqn (2) & (3), $A = B = I$,

from (1) $i_c(t) = -I + 2I\left[\dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]$.

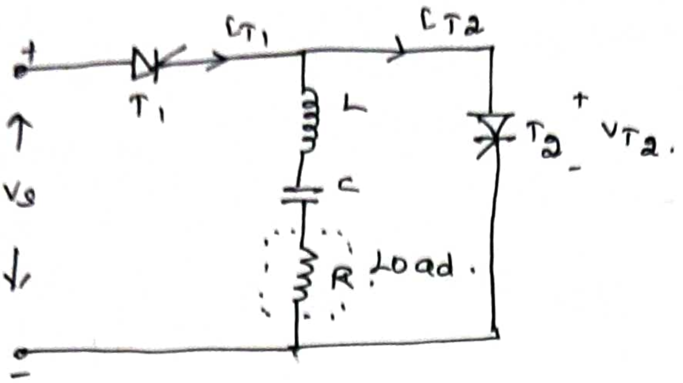
$$i_c(t) = I\,[2\cos\omega_0 t - 1].$$

$$v_c(t) = \frac{2I}{\omega_0 C}\sin\omega_0 t.$$



SERIES INVERTER :-

Inverters in which commutating components are permanently connected in series with the load are called series inverter. The series circuit must be underdamped. series inverter also called load commutated inverters or self commutated inverters.



=) It consists of load resistance R, in series with commutating components L and C.

=) When T_1 is turned ON, T_2 off, i starts building up in the RLC circuit.

=) The load current after reaching peak value, decays to zero at a point a.

=) at pt a, load current tends to reverse.

=) minimum time is given by,

$$t_{q,min} = \frac{\pi}{\omega} - \frac{\pi}{\omega_r} = \frac{1}{2}\left(\frac{1}{f} - \frac{1}{f_r}\right).$$

$\omega \rightarrow$ output frequency r/s.

$\omega_r \rightarrow$ circuit ringing frequency in r/s.

$T_1 \rightarrow$ OFF, $T_2 \rightarrow$ ON, $T_{off} > t_{q,min}$.

$C \rightarrow$ discharge, load current builds up in the reverse direction, to some peak negative value and decays to zero.

After this time $T_{off} = cd$ must elaps for T_2 to recover. At d, T_1 is again turned on. The process repeats.

Analysis of Basic series Inverter :-

when T_1 is turned on,

$$Ri + L \cdot \frac{di}{dt} + \frac{1}{C}\int i\,dt = V_s. \quad \ldots (1)$$

with zero initial conditions, L.T is,

$$I(s)\left[R + Ls + \frac{1}{sC}\right] = \frac{V_s}{s}.$$

$$I(s) = \frac{V}{L} \cdot \frac{1}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} \quad \ldots (2).$$

root of $s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$ are

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}.$$

The circuit is underdamped,

$\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ must be negative.

$$\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} < 0, \quad R^2 < \frac{4L}{C}.$$

$$s = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

$$s = -\xi + j\omega_r.$$

$$\xi = -R/2L, \quad \omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

If $\omega_0 = \frac{1}{\sqrt{LC}}$, $\omega_r = \sqrt{\omega_0^2 - \xi^2}$

$$\omega_0 = \sqrt{\omega_r^2 + \xi^2}.$$

from (2),

$$I(s) = \frac{V_s}{L}\left[\frac{1}{(s+\xi-j\omega_r)(s+\xi+j\omega_r)}\right].$$

Let $\frac{1}{(s+\xi-j\omega_r)(s+\xi+j\omega_r)} = \frac{A}{s+\xi-j\omega_r} + \frac{B}{s+\xi+j\omega_r}$.

$$A = \frac{1}{2j\omega_r} \; ; \quad B = -\frac{1}{2j\omega_r}.$$

$$I(s) = \frac{V_s}{L}\cdot\frac{1}{\omega_r}\left[\frac{\omega_r}{(s+\xi)^2+\omega_r^2}\right].$$

inverse Laplace Transform is,

$$i(t) = \frac{V_s}{\omega_r . L} e^{-\xi t} \sin\omega_r t. \quad \ldots (3).$$

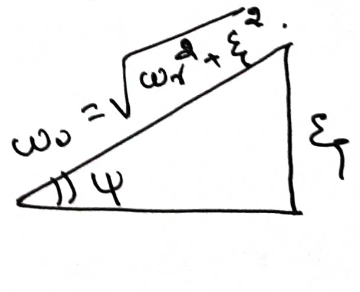
Ringing frequency $f_r = \frac{1}{2\pi}\cdot\sqrt{\frac{1}{LC} - (R/2L)^2}$ Hz, $f < f_r$.

From eqn (3),

$$V_L = L.\frac{di}{dt} = L.\frac{V_s}{L}\cdot\frac{1}{\omega_r}\left[e^{-\xi t}.\omega_r\cos\omega_r t - \xi.e^{-\xi t}\sin\omega_r t\right].$$

$$V_L = V_s.\frac{\omega_0}{\omega_r} e^{-\xi t}.\cos(\omega_r t+\varphi).$$

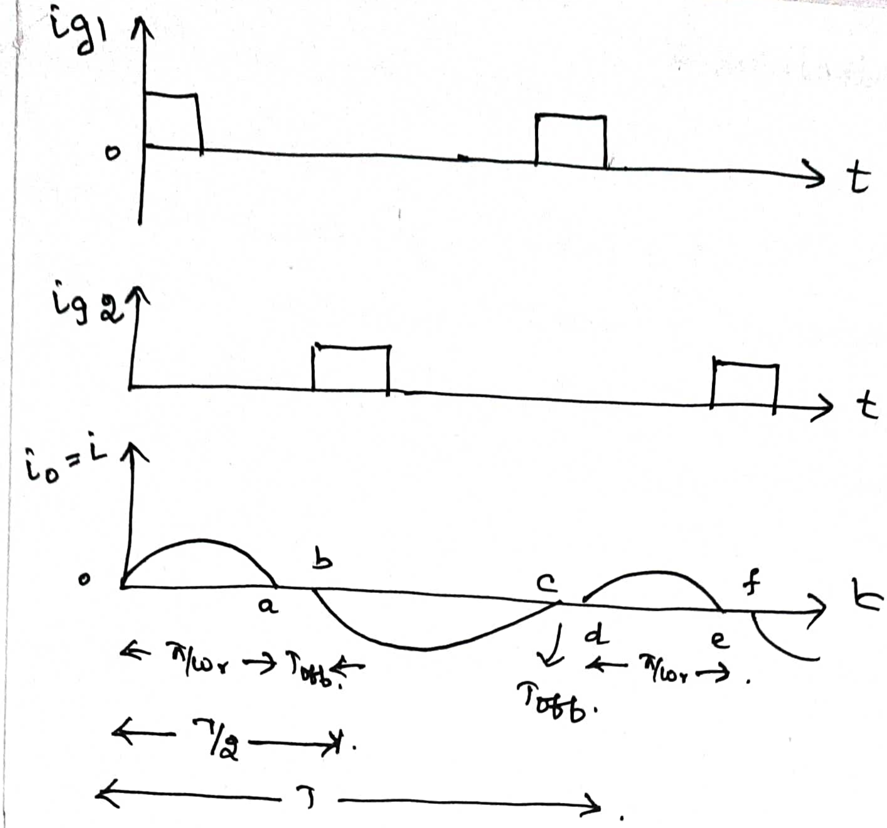
$\omega_0 \rightarrow$ resonant frequency.



$$\omega_0 = \sqrt{\omega_r^2+\xi^2},$$

$$\varphi = \tan^{-1}\left(\frac{\xi}{\omega_r}\right).$$

$$V_C = V_s\left[1 - e^{-\xi t}\frac{\omega_0}{\omega_r}.\cos(\omega_r t - \varphi)\right].$$



Load current waveform for basic series inverter.

comparison of VSI & CSI :-

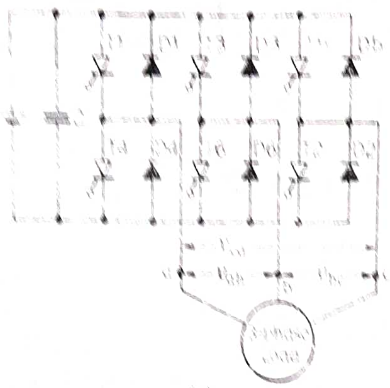
VSI	CSI
1) VSI is fed from a DC voltage source having small impedance.	CSI is fed from DC voltage source of high impedance.
2) input voltage maintained constant	Input is constant but adjustable.
3) output voltage doesnot dependent on the load.	amplitude of current independent of load.
4) VSI requires feedback diodes.	feed back diodes not required.
5) commutation circuit is complicated.	commutation circuit simple.

UNIT - III

Inverters.

Single phase and three phase voltage source inverters (both 120° mode and 180° mode) - Voltage & harmonic control - PWM techniques : sinusoidal PWM, modified sinusoidal PWM - multiple PWM - introduction to space vector modulation - current source inverter.

Three phase voltage source inverter (180° mode) :-



(a)

Three phase inverter is a six step bridge inverter. It uses a minimum of 6 thyristor. A step is defined as a change in the firing from one thyristor to the next thyristor in proper sequence. A large capacitor is used to make the input dc voltage constant.

T_1 triggered at $\omega t = 0°$, and conducts for $180°$.

T_2 triggered at $\omega t = 60°$, and conducts for $60° + 180° = 240°$

T_3 triggered at $\omega t = 120°$, conducts for $120° + 180° = 300°$

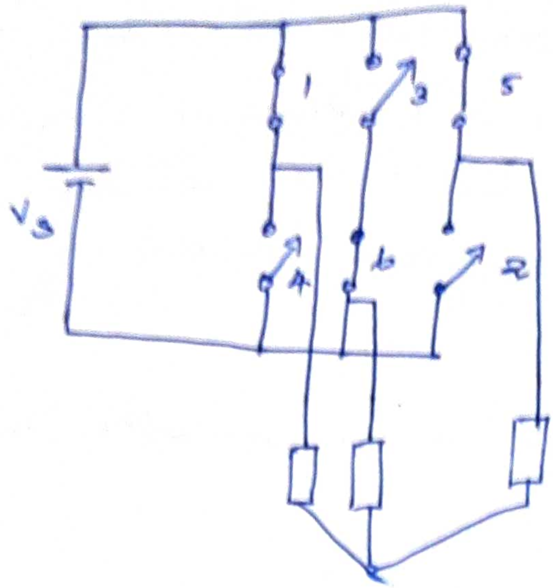
T_4 triggered at $\omega t = 180°$, conducts for $180° + 180° = 360°$.

T_5 triggered at $\omega t = 240°$, conducts for $240 + 180° =$ ~~300~~ $420°$.

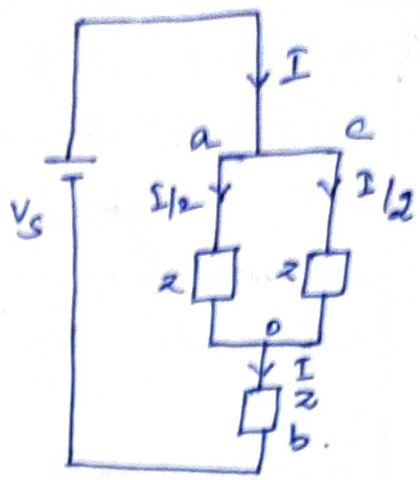
T_6 triggered at $\omega t = 300°$, conducts for $300 + 180° = 480°$.

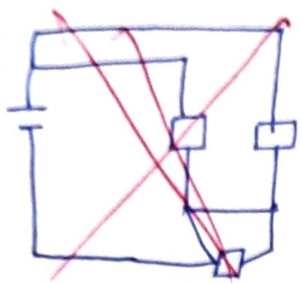
Equivalent circuit :

Mode I : 0 – 60°, 5, 6, 1 are conduct



⇒





Total impedance,

$$z \parallel z + z = \frac{z \times z}{z+z} + z$$

$$= \frac{z^2}{2z} + z$$

$$= \frac{3z^2}{2z}.$$

$$\boxed{Z_{equ} = \frac{3z}{2}}$$

$$I = \frac{V_s}{z} = \frac{V_s}{\frac{3z}{2}} = 2$$

$$V_{ao} = I/2 \times z = \frac{\cancel{2}V_s}{\cancel{2}\times \cancel{3}\cancel{z}} \times \frac{\cancel{3z}}{\cancel{2}}$$

$$= \frac{V_s}{3}.$$

$$V_{bo} = -I \times z = -\frac{2V_s}{3z} \times z = -\frac{2V_s}{3}.$$

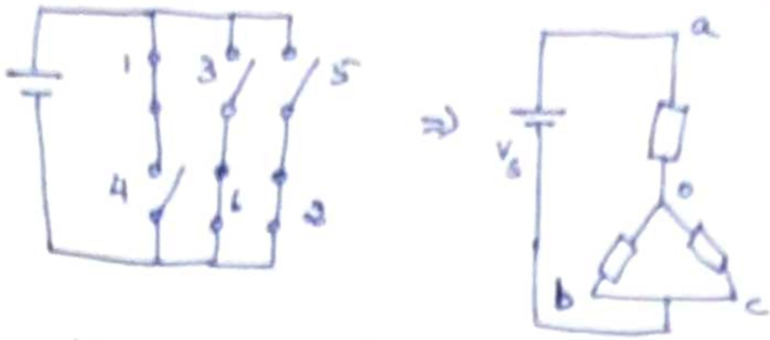
$$V_{co} = I/2 \times z = \frac{2V_s}{3z} \times z/2 = V_s/3.$$

$$V_{ab} = V_{ao} + V_{ob} = V_s/3 + \frac{2V_s}{3} = V_s.$$

$$V_{bc} = V_{bo} + V_{oc} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s.$$

$$V_{ca} = V_{co} + V_{oa} = V_s/3 - V_s/3 = 0.$$

Mode (ii) $60^\circ - 120^\circ$, 6, 1, 2 are conduct



$V_{ao} = \frac{2V_s}{3}$

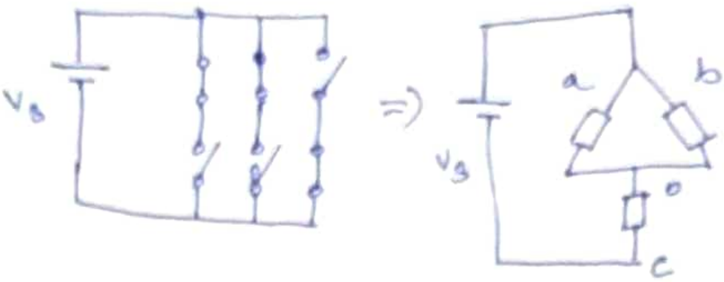
$V_{ob} = \frac{V_s}{3}$

$V_{oc} = V_s/3$

$V_{ab} = V_{ao} + V_{ob} = V_s$; $V_{bc} = V_{bo} + V_{oc} = -\frac{V_s}{3} + \frac{V_s}{3} = 0$

$V_{ca} = V_{co} + V_{oa} = -\frac{V_s}{3} - \frac{2V_s}{3} = -V_s$

Mode (iii) $120^\circ - 180^\circ$, 1, 2, 3 are conduct.



$V_{ao} = \frac{V_s}{3}$

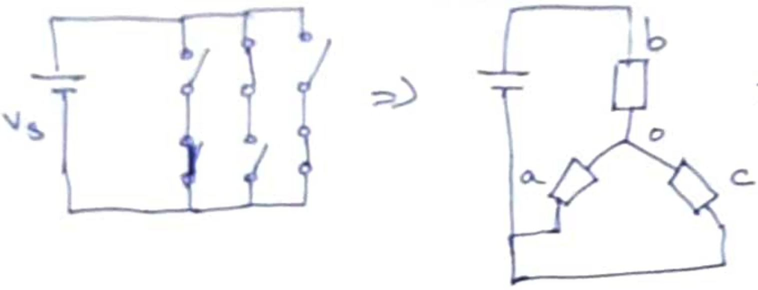
$V_{bo} = \frac{V_s}{3}$

$V_{oc} = +\frac{2V_s}{3}$

$V_{ab} = V_{ao} + V_{ob} = 0$; $V_{bc} = V_{bo} + V_{oc} = V_s$;

$V_{ca} = V_{co} + V_{oa} = -V_s$.

Mode (iv) ; $180^\circ - 240^\circ$, 2, 3, 4 conduct.



$V_{bo} = \frac{2V_s}{3}$

$V_{oa} = V_s/3$.

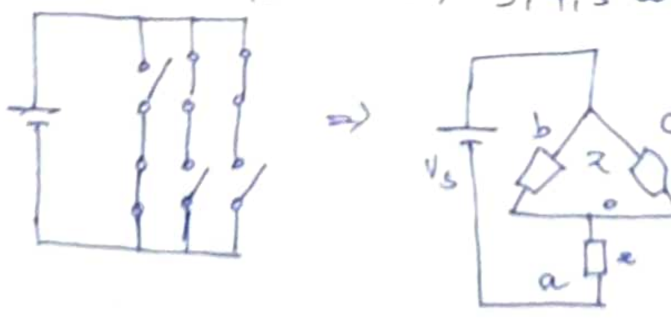
$V_{oc} = V_s/3$.

$V_{ab} = V_{ao} + V_{ob} = -\frac{V_s}{3} - \frac{2V_s}{3} = -V_s$.

$V_{bc} = V_{bo} + V_{oc} = \frac{2V_s}{3} + V_s/3 = V_s$.

$V_{ca} = V_{co} + V_{oa} = -V_s/3 + V_s/3 = 0$.

Mode (v) ; $240^\circ - 300^\circ$, 3, 4, 5 are conduct



$V_{bo} = V_s/3$.

$V_{co} = V_s/3$.

$V_{oa} = \frac{2V_s}{3}$

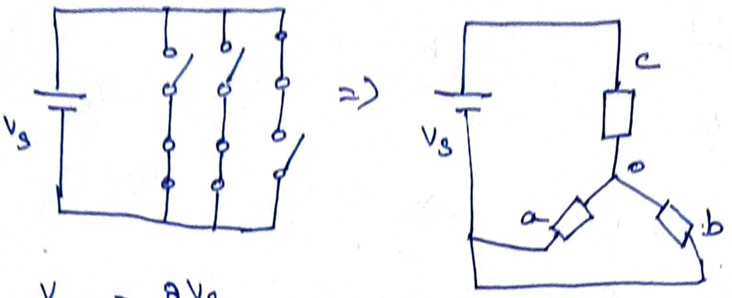
$V_{ab} = V_{ao} + V_{ob}$

$= -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s$.

$V_{bc} = V_{bo} + V_{oc} = \frac{V_s}{3} + \left(-\frac{V_s}{3}\right) = 0,$

$V_{ca} = V_{co} + V_{oa} = \frac{V_s}{3} + \frac{2V_s}{3} = V_s.$

Mode (vi) :- $300° - 360°$, 4, 5, 6 are conduct.

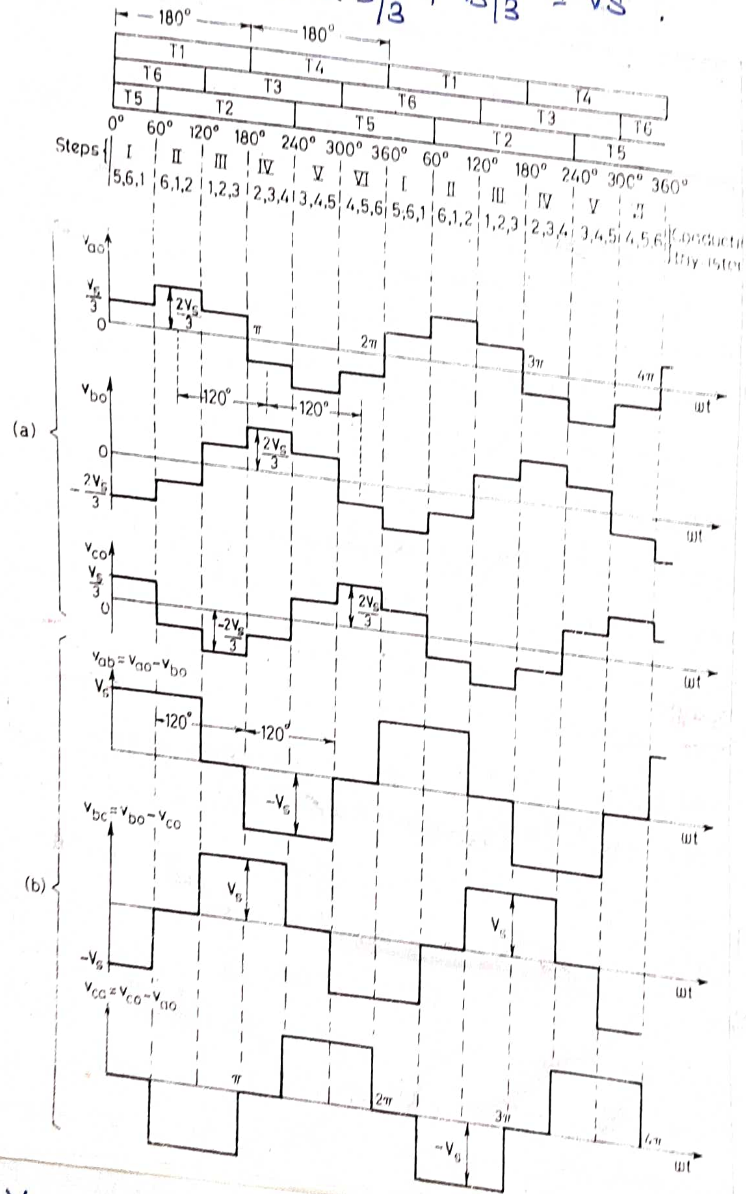


$V_{co} = \frac{2V_s}{3}$; $V_{oa} = \frac{V_s}{3}$; $V_{ob} = V_s/3$.

$V_{ab} = V_{ao} + V_{ob} = -V_s/3 + V_s/3 = 0;$

$V_{bc} = V_{bo} + V_{oc} = -V_s/3 - 2V_s/3 = -V_s.$

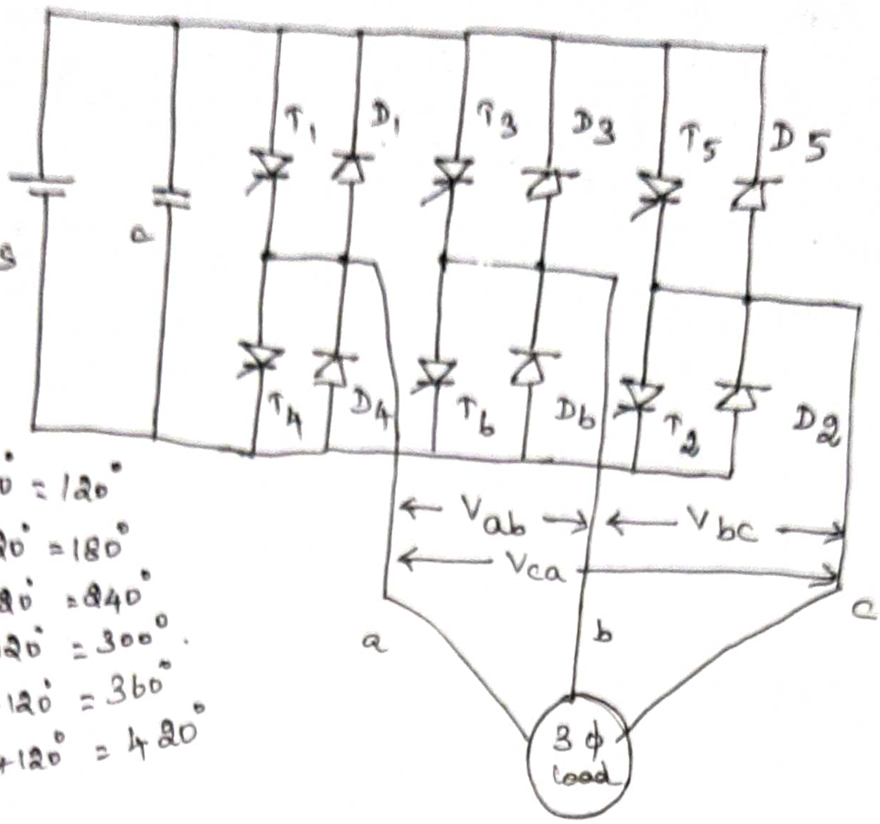
$V_{ca} = V_{co} + V_{oa} = 2V_s/3 + V_s/3 = V_s.$



$V_{bc} = V_{bo} + V_{oc} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s.$

$V_{ca} = V_{co} + V_{oa} = V_s/3 - V_s/3 = 0.$

120° mode with star connected inverter (3 φ).



$T_1 = 0° + 120° = 120°$

$T_2 = 60° + 120° = 180°$

$T_3 = 120° + 120° = 240°$

$T_4 = 180° + 120° = 300°$

$T_5 = 240° + 120° = 360°$

$T_6 = 300° + 120° = 420°$

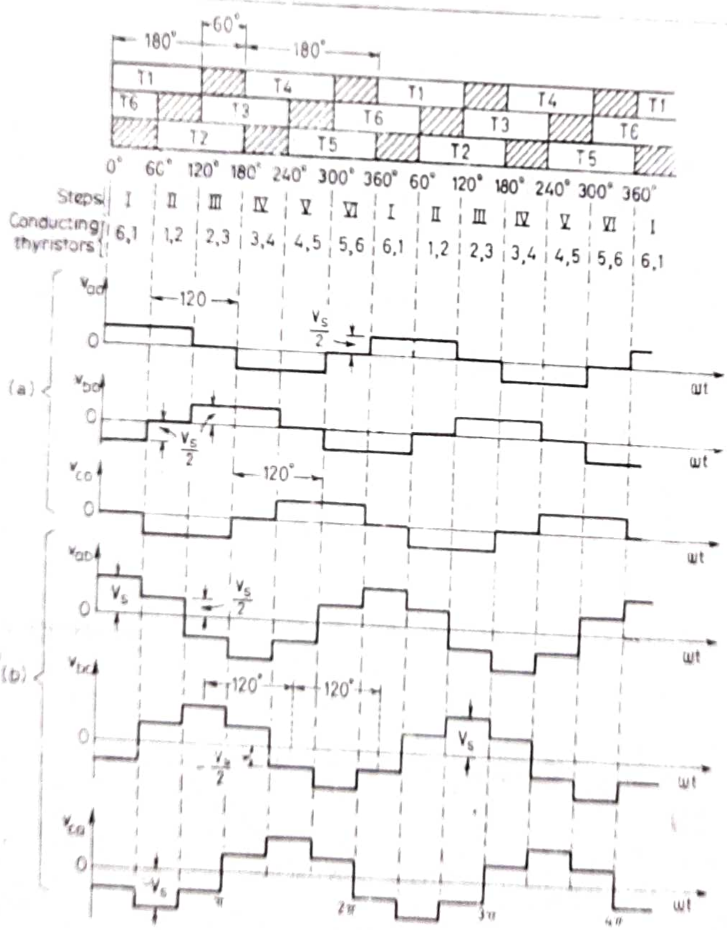
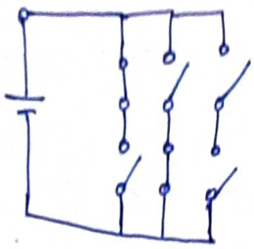
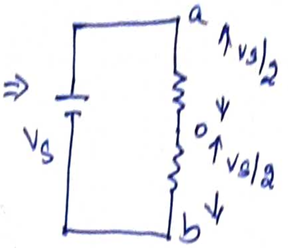


Fig. 8.22. Voltage waveforms for 120° mode six-step 3-phase VSI.

step I ; 0-60°, 6, 1 closed



$\Rightarrow$



$V_{ao} = V_s/2$

$V_{ob} = V_s/2$.

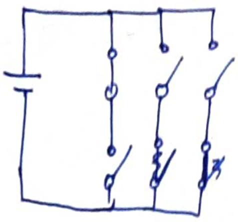
$V_{oc} = 0$.

$V_{ab} = V_{ao} + V_{ob} = V_s$

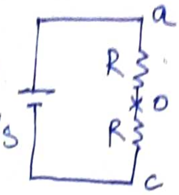
$V_{bc} = V_{bo} + V_{oc} = -V_s/2$.

$V_{ca} = V_{co} + V_{oa} = -V_s/2$.

step II : 60°-120°, 1, 2 closed



$\Rightarrow$



$V_{ao} = V_s/2$

$V_{oc} = V_s/2$.

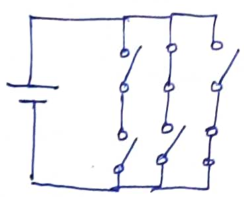
$V_{bo} = 0$.

$V_{ab} = V_{ao} + V_{ob} = V_s/2$.

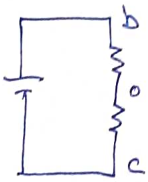
$V_{bc} = V_{bo} + V_{oc} = 0 + V_s/2 = V_s/2$

$V_{ca} = V_{co} + V_{oa} = -V_s/2 - V_s/2 = -V_s$.

step III :- 120° - 180°, 2, 3 closed.



$\Rightarrow$



$\Rightarrow$ $V_{ao} = 0$.

$V_{bo} = V_s/2$

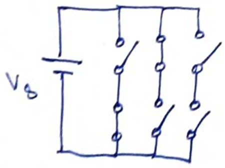
$V_{oc} = V_s/2$.

$V_{ab} = V_{ao} + V_{ob}$. $= -V_s/2$

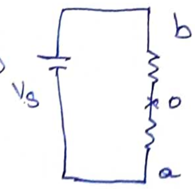
$V_{bc} = V_{bo} + V_{oc} = V_s/2 + V_s/2 = V_s$

$V_{ca} = V_{co} + V_{oa} = -V_s/2$

step iv :- 180° - 240°, 3, 4 closed.



$\Rightarrow$



$\Rightarrow$ $V_{bo} = V_s/2$.

$V_{oa} = V_s/2$

$V_{oc} = 0$.

$V_{ab} = V_{ao} + V_{ob} = -V_s/2 - V_s/2 = -V_s$.

$V_{bc} = V_{bo} + V_{oc} = V_s/2$

$V_{ca} = V_{co} + V_{oa} = 0 + V_s/2 = V_s/2$.

ɔal

ws

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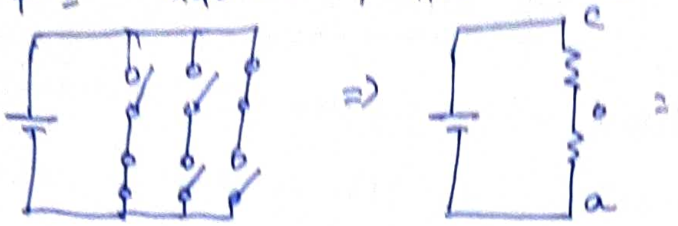
$V_{bo} + V_{oc}$

$V_{ca} = V_{co} + V_{oa}$ $= -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s$.

$= V_s/3 - V_s/3 = 0$.

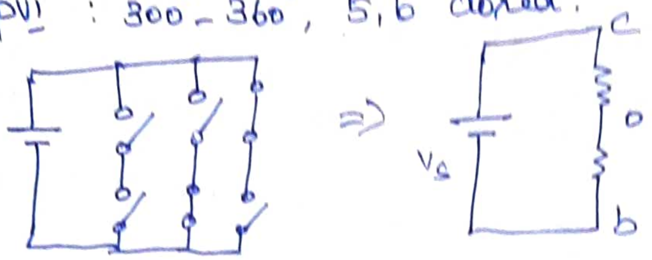
Step $\bar{V}$: $240^\circ - 300^\circ$, 4, 5 closed.



$V_{co} = V_s/2$
$V_{oa} = V_s/2$
$V_{ob} = 0$.

$V_{ab} = V_{ao} + V_{ob} = -V_s/2$
$V_{bc} = V_{bo} + V_{oc} = 0 - V_s/2 = -V_s/2$
$V_{ca} = V_{co} + V_{oa} = V_s/2 + V_s/2 = V_s$.

Step $\bar{VI}$: $300^\circ - 360^\circ$, 5, 6 closed.



$V_{co} = V_s/2$
$V_{ob} = V_s/2$.
$V_{ao} = 0$.

$V_{ab} = V_{ao} + V_{ob} = V_s/2$; $V_{bc} = V_{bo} + V_{oc} = -V_s/2 - V_s/2 = -V_s$.
$V_{ca} = V_{co} + V_{oa} = V_s/2 + 0 = V_s/2$.

Fourier analysis of phase voltage waveform,

$$V_{ao} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n(\omega t + \pi/6).$$

$$V_{bo} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n(\omega t - \pi/2).$$

$$V_{co} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n(\omega t + 5\pi/6).$$

$$V_{ab} = \sum_{n=6k\pm1}^{\infty} \frac{3V_s}{n\pi} \sin n(\omega t + \pi/3).$$

$k = 0, 1, 2, 3 \ldots$

Step I ; 0-60°, 6, 1 closed

Voltage control in 1ϕ inverter :-

An ac load may require a constant input voltage. Any variations in the dc input voltage must be compensated in order to maintain a constant voltage at the a.c load terminals.

The various methods for the control of output voltage of inverters are as

(i) External control of ac output voltage

(ii) External control of dc input voltage

(iii) Internal control of inverter.

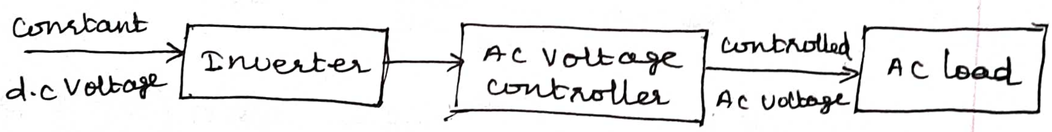
External control of a.c output voltage :-

There are two possible methods. They are

(i) AC voltage control

(ii) Series - inverter control.

(i) AC voltage control :-



The voltage input to ac load is regulated through the firing angle control of ac voltage controller. This method gives rise to higher harmonic content in the output voltage.

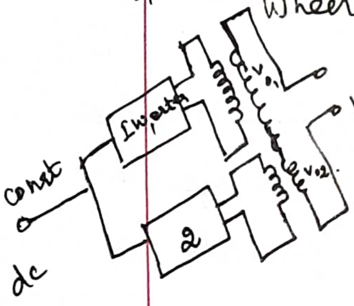
$V_S/3 - V_S/3 = 0$.

(b) Series - Inverter control :-

In this method, the inverter output is fed to two transformers whose secondaries are connected in series. Phasor sum of the two fundamental voltages V_{01}, V_{02} gives the resultant fundamental voltage V_0. Here V_0 is given by,

$$V_0 = \left[V_{01}^2 + V_{02}^2 + 2\,V_{01} \cdot V_{02} \cdot \cos\theta\right]^{1/2}.$$

When θ is zero,



$$= \left[V_{01}^2 + V_{02}^2 + 2\,V_{01} \cdot V_{02}\right]^{1/2} \quad \therefore [\cos 0 = 1]$$

$$= \left[(V_{01} + V_{02})^2\right]^{1/2}$$

$$= V_{01} + V_{02}.$$

when $\theta = \pi$, $= V_0 = 0$. in case $V_{01} = V_{02}$.

The angle θ can be varied by the firing angle control of two inverters.

(2) External control of dc input voltage :-

constant ac voltage → [Fully controlled rectifier] → [filter] → controlled dc voltage → [Inverter] → Controlled ac voltage.

constant ac voltage → [uncontrolled rectifier] → [chopper] → [filter] → controlled dc → [inverter] → Controlled ac voltage

constant ac voltage → [AC voltage Controller] → [uncontrolled rectifier] → [filter] → Controlled dc voltage → [inverter] → controlled ac voltage.

constant dc voltage → [Chopper] → [filter] → controlled dc voltage → [Inverter] → Controlled ac voltage.

step I ; 0-60°, 6, 1 closed

Voltage control in 1ϕ inverter :-

Disadvantages :-

(i) Number of power converters are increased from two to three.

(ii) For reducing ripple ~~current~~ content, filter circuit is required. This increase the cost, weight and size.

(iii) Dc input decreased, commutating capacitor voltage also decreases.

(3) Internal control of Inverter :-

Pulse width modulation control :-

A fixed dc input voltage is given to the inverter and a controlled ac output voltage is obtained by adjusting the ON and OFF periods of the inverter components.

Advantages :-

(i) Does not require any additional components.

(ii) low order harmonics can be eliminated, filtering requirements are minimized.

Disadvantages :-

(i) SCRs are expensive, they must possess low turn ON, turn off times.

~~(ii)~~

Harmonic Elimination and reduction in harmonics by PWM :-

2) n^{th} harmonic can be eliminated by a proper choice of displacement angle β.

$$\sin n\beta/2 = 0.$$

$$\beta = 360/n.$$

3^{rd} harmonic will be eliminated if,

$$\beta = \frac{360}{3} = 120^\circ.$$

The fourier series of output voltage can be expressed as,

$$V_0 = \sum_{n=1,3,5}^{\infty} A_n \sin n\omega t$$

$$A_n = \frac{4V_s}{n\pi}\left[\int_0^{\alpha_1} \sin n\omega t \cdot d(\omega t) - \int_{\alpha_1}^{\alpha_2} \sin n(\omega t)\cdot d\omega t + \int_{\alpha_2}^{\alpha} \sin n\omega t \cdot d(\omega t)\right].$$

$$= \frac{4V_s}{\pi}\left[\frac{1-2\cos n\alpha_1 + 2\cos n\alpha_2}{n}\right]$$

The 3^{rd} & 5^{th} harmonics would be eliminated, if A3.

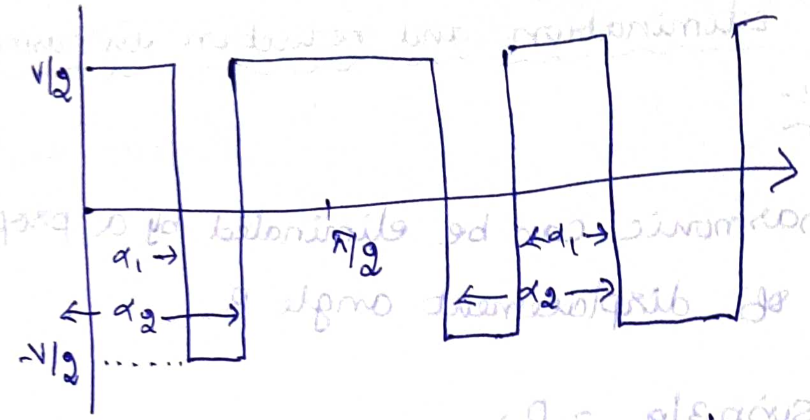
$$1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2 = 0.$$

$$\alpha_1 = 1/3 \cos^{-1}(\cos 3\alpha_1 - 0.5) \quad \ldots\ldots (1).$$

$$1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 = 0.$$

$$\alpha_1 = 1/5 \cos^{-1}(\cos 5\alpha_2 + 0.5) \quad \ldots\ldots (2).$$

step I ; 0-60°, 6, 1 closed

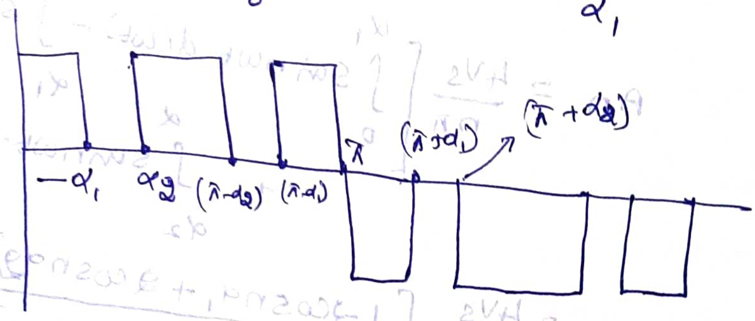


Equation (1) & (2) can be solved iteratively by assuming $\alpha_1 = 0$. repeating the calculations for α_1, α_2. The result is $\alpha_1 = 23.62^\circ$, $\alpha_2 = 33.3^\circ$

$$A_n = \frac{4V_s}{n\pi}\left(1 - 2\cos n\alpha_1 + 2\cos n\alpha_2 - 2\cos n\alpha_3 + 2\cos n\alpha_4 - \right) \ldots\ldots (3)$$

with unipolar notches :-

$$A_n = \frac{4V_s}{\pi}\int_0^{\alpha_1} \sin n\omega t \cdot d\omega t + \int_{\alpha_1}^{\pi/2} \sin n\omega t \cdot d\omega t.$$



$$= \frac{4V_s}{n\pi}\left(1 - \cos n\alpha_1 + \cos n\alpha_2\right).$$

5th, 3rd harmonics will be eliminated

$$1 - \cos 3\alpha_1 + \cos 3\alpha_2 = 0.$$

$$1 - \cos 5\alpha_1 + \cos 5\alpha_2 = 0.$$

$$\alpha_1 = 17.83,\ \alpha_2 = 37.97^\circ;$$

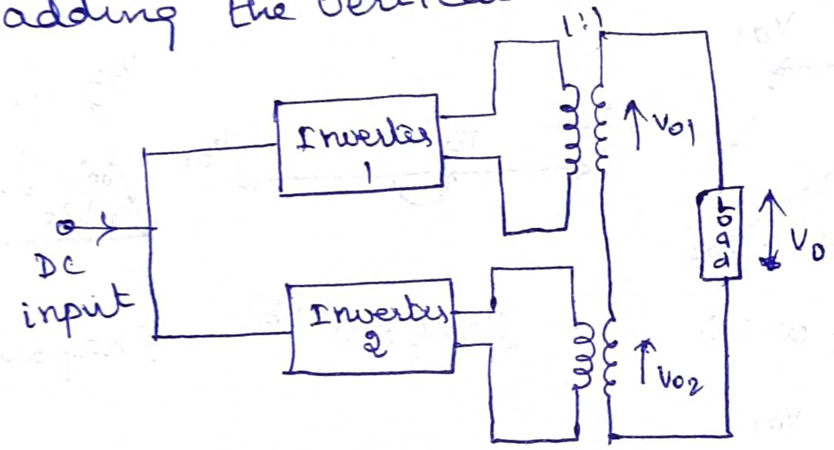
Harmonic Reduction by Transformer Connections :-

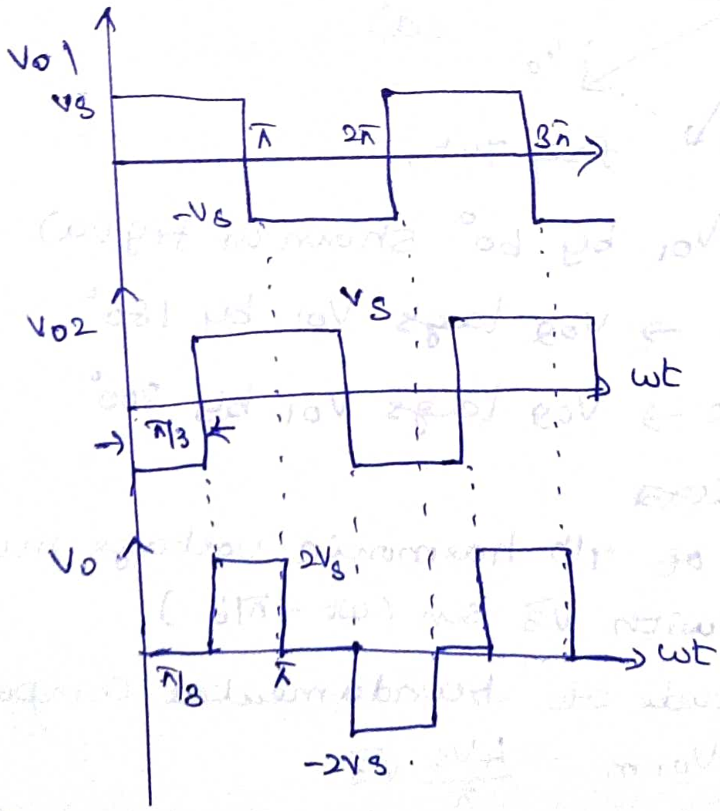
Output voltage from two or more inverters can be combined by means of transformers to get a net output voltage with reduced harmonic content. Two transformers are in series.

V_{o1} → from inverter 1

V_{o2} → from inverter 2.

V_{o2} have a phase shift of $\pi/3$ radians with respect to V_{o1}. The resultant voltage V_o obtained by adding the vertical coordinates of V_{o1} & V_{o2}.





step I ; 0-60°, 6, 1 closed

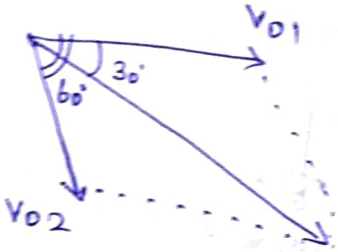
$$V_{o1} = \frac{4V_s}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\omega t \ldots\right]$$

$$V_{o2} = \frac{4V_s}{\pi}\left[\sin(\omega t - \pi/3) + \frac{1}{3}\sin 3(\omega t - \pi/3) + \frac{1}{5}\sin 5(\omega t - \pi/3) + \frac{1}{7}\sin 7(\omega t - \pi/3)\right]$$

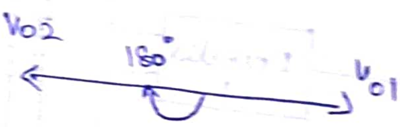
resultant voltage V_o is,

$$V_o = V_{o1} + V_{o2}$$

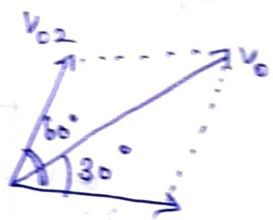
$$= \frac{4V_s}{\pi}\sqrt{3}\left[\sin(\omega t - \pi/6) + \frac{1}{5}\sin(5\omega t + \pi/6) + \frac{1}{7}\sin(7\omega t - \pi/6) + \ldots\right].$$



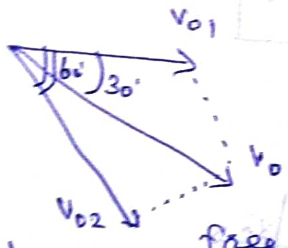
freq ω (a)



freq 3ω (b)



freq 5ω (c)



(d). freq 7ω.

fundamental frequency ⇒ V_{o2} lags V_{o1} by 60° shown in fig (a).

3rd harmonic → V_{o2} lags V_{o1} by 180°.

5th harmonic → V_{o2} lags V_{o1} by 300°.

~~7th harmonics~~

resultant of 7th harmonic voltage must be associated with $\sqrt{3}\sin(\omega t - \pi/6)$.

The amplitude of fundamental component of V_o,

$$V_{o1m} = \frac{4V_s}{\pi}\sqrt{3}.$$

Space Vector Modulation :-

Space vector approach to PWM involves the use of voltage space vectors as reference, instead of 3 ϕ modulating waves considers combined effect of all 3 ϕ voltages.

At steady state the voltage space vector has a constant magnitude and revolves with constant frequency the direction of rotation depends on the phase sequence.

SVPWM is used for inverter fed drives because of its superior harmonic quality and extended linear range of operation.

If a 3 ϕ windings displaced in space by 120° are excited by 3 ϕ currents with a phase difference of 120° a magnetic field rotating in space will be generated.

Line to line voltages

$V_{ab} = V_{aN} - V_{bN}$

$V_{bc} = V_{bN} - V_{cN}$

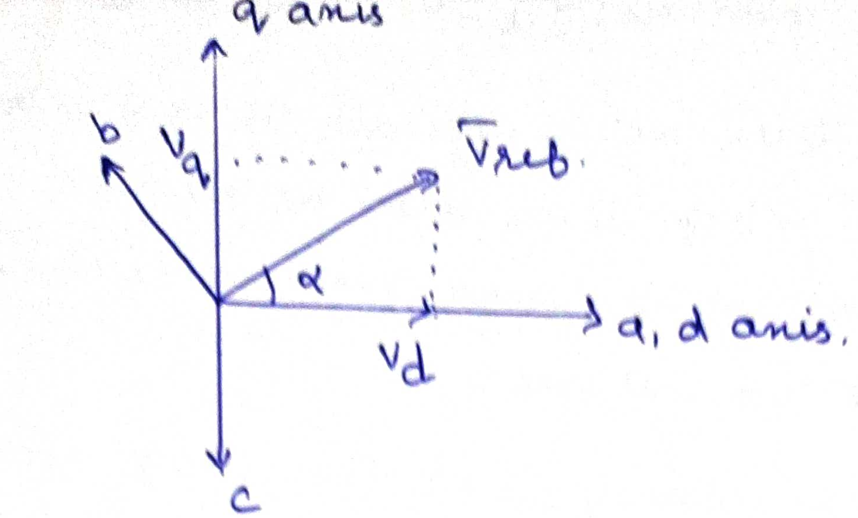
$V_{ca} = V_{cN} - V_{aN}$

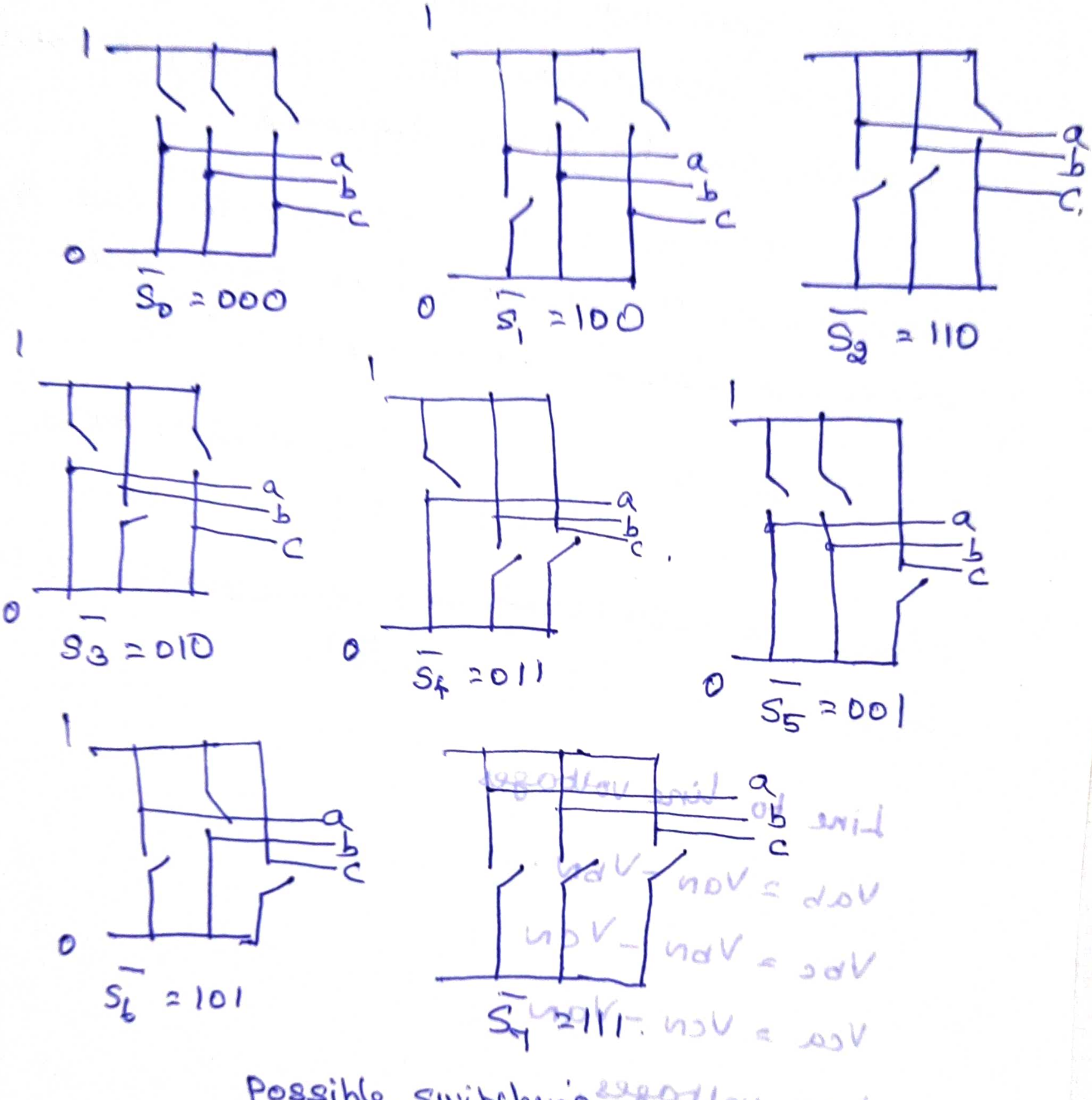
phase voltages

$V_{an} = \frac{2}{3} V_{aN} - \frac{1}{3} V_{bN} - \frac{1}{3} V_{cN}$

$V_{bn} = -\frac{1}{3} V_{aN} + \frac{2}{3} V_{bN} - \frac{1}{3} V_{cN}$

$V_{cn} = -\frac{1}{3} V_{aN} - \frac{1}{3} V_{bN} + \frac{2}{3} V_{cN}$





Possible switching.

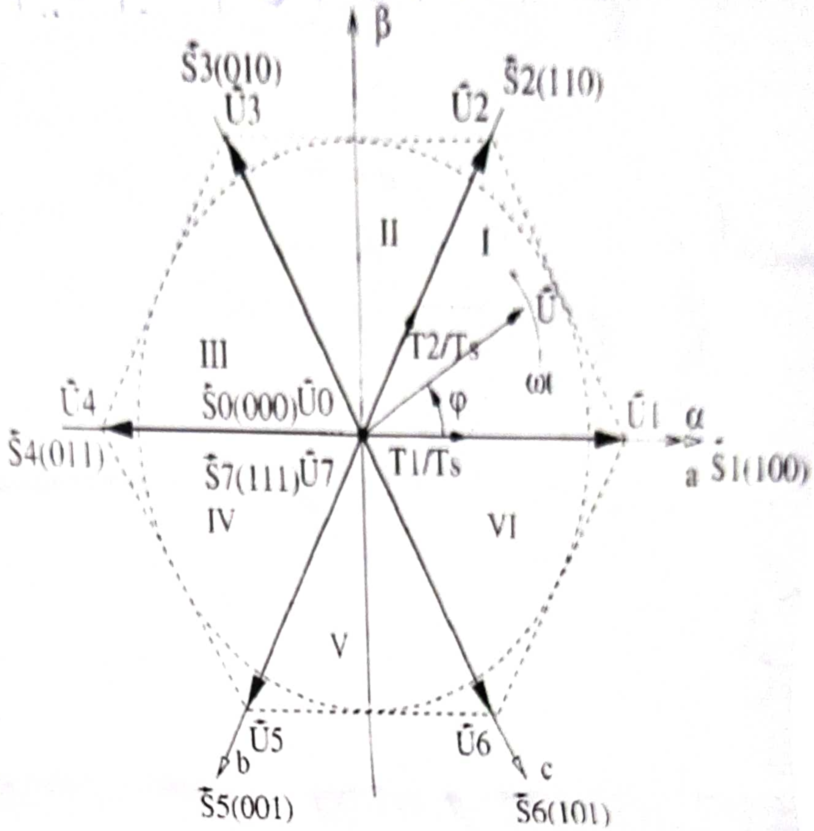
State	Vector
S_1	100
S_2	110
S_3	010
S_4	011
S_5	001
S_6	101
S_7	111
S_0	000

In the vector spacing, according to the equivalence principle, the following operation rules are obeyed :

$U_1 = -U_4$

$U_2 = -U_5$

$U_3 = -U_6$, $U_7 = U_0 = 0$, $U_1 + U_3 + U_5 = 0$.



In one sampling interval, the output voltage vector can be written as,

$$\vec{U}(t) = \frac{t_0}{T_s}\vec{U_0} + \frac{t_1}{T_s}\vec{U_1} + \dots + \frac{t_0}{T_s}\vec{U_7}.$$

The decomposition of U into $U_1, U_2, U_3, U_4, U_5, U_6, U_7$ has infinite ways. In order to reduce the number of switching actions and make full use of active turn on time for space vectors, vector U can be expressed as,

$$\vec{U} = \frac{T_1}{T_s}\vec{U_1} + \frac{T_1}{T_s}\vec{U_2} + \frac{T_7}{T_s}\vec{U_7} + \frac{T_0}{T_s}\vec{U_0}.$$

where $T_s - T_1 - T_2 = T_0 + T_7 \geq 0$,

$T_7 \geq 0$ & $T_0 \geq 0$.

Voltage vectors	Switching vectors			Line to neutral voltage			Line to line voltage		
	a	b	c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
V_0	0	0	0	0	0	0	0	0	0
V_1	1	0	0	2/3	-1/3	-1/3	1	0	-1
V_2	1	1	0	1/3	1/3	-2/3	0	1	-1
V_3	0	1	0	-1/3	2/3	-1/3	-1	1	0
V_4	0	1	1	-2/3	1/3	1/3	-1	0	1
V_5	0	0	1	-1/3	-1/3	2/3	0	-1	1
V_6	1	0	1	1/3	-2/3	1/3	1	-1	0
V_7	1	1	1	0	0	0	0	0	0

~~Applications~~ :- Realization of Space Vector PWM:-

Step 1 : Determine V_d, V_q, V_{ref} and angle.

Step 2 : Determine time duration T_1, T_2, T_0.

Step 3 : Determine the switching time of each transistor (S_1 to S_6).

Co-ordinate transformation : abc to dq.

Voltage control of single phase inverter :-

i) Single pulse width modulation.

ii) Multiple pulse width modulation.

iii) Sinusoidal pulse width modulation.

iv) Modified sinusoidal PWM.

v) Phase displacement control.

The methods are applicable to 3φ inverter.

i) Single pulse width modulation :-

Only one pulse per half cycle and the output rms voltage is changed by varying the width of the pulse. The gating signals are generated by comparing the rectangular control signal of amplitude A_r with triangular carrier signal A_c.

$$\text{Modulation index } M = \frac{A_r}{A_c}.$$

$$\text{RMS value of output voltage } V_{or} = \left[\frac{1}{\pi}\int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_s^2\, d(\omega t)\right]^{1/2}$$

Fourier series of output voltage,

$$V_o = \sum_{1,3,5} (A_n \cos n\omega t + B_n \sin n\omega t).$$

Half wave symmetry, $a_0 = a_n = 0$.

$$B_n = \frac{2}{\pi}\int_0^{\pi} V_s \sin\omega t \, d\omega t = \frac{2}{\pi}\int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_s \sin\omega t \, d\omega t.$$

$$= \frac{2V_s}{\pi}\left(\frac{-\cos n\omega t}{n}\right)_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} = \frac{2V_s}{n\pi}(\cos n\omega t)_{\frac{\pi+\delta}{2}}^{\frac{\pi-\delta}{2}}.$$

$$= \frac{2V_s}{n\pi}\left[\cos n\left(\frac{\pi-\delta}{2}\right) - \cos n\left(\frac{\pi+\delta}{2}\right)\right].$$

step I ; 0-60°, 6, 1 closed

$$V_o = \sum_{1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin \frac{n\delta}{2} \sin n\omega t .$$

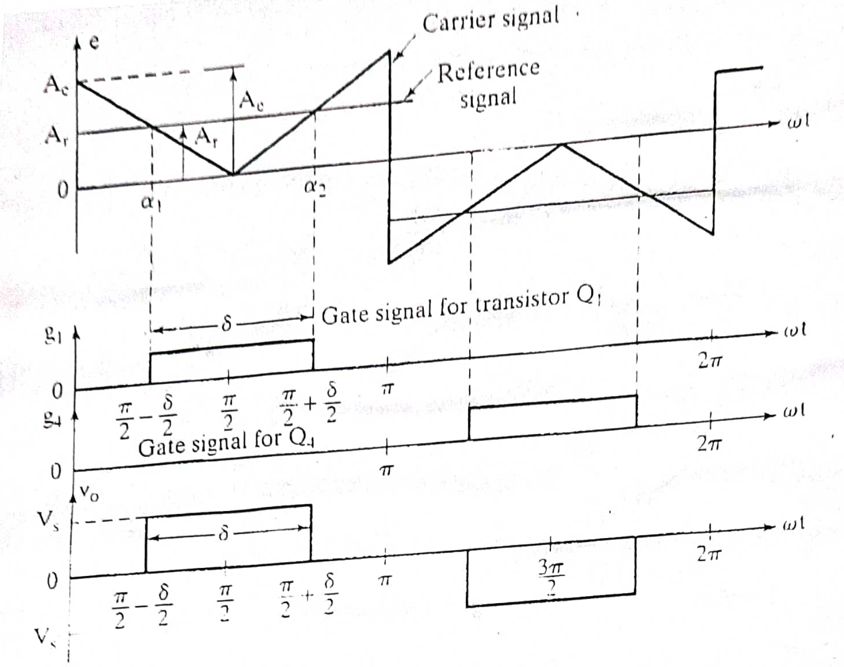


Fig 1.7 Single-Pulse-Width-Modulation

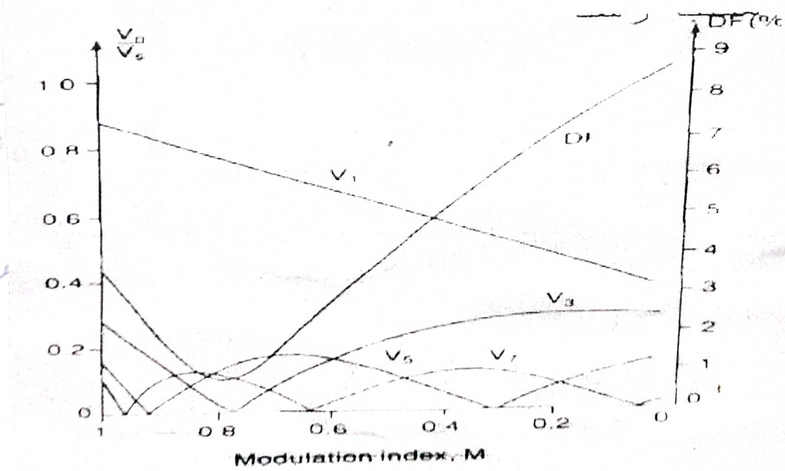


Fig 1.8 Harmonic profile

Sb ST

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Multiple Pulse width Modulation :-

In multiple PWM control, instead of having a single pulse per half cycle, there will be multiple number of pulses per half cycle all of them being of equal width.

$f_0 = f_r$ ∴ output frequency is determined by frequency of reference signal.

f_c determines no. of pulses / half cycle.

No. of pulses / half cycle $= p = \frac{f_c}{2f_0} = \frac{m_f}{2}$.

$m_f \rightarrow$ frequency modulation ratio.

$M \rightarrow$ varied from 0 to 1

pulse width 0 to π/p.

voltage 0 to V_s.

output RMS voltage $V_{or} = \left[\frac{1}{\pi/p} \int_{(\frac{\pi}{p} - \delta)/2}^{(\pi/p + \delta)/2} V_s^2 \, d\omega t \right]^{1/2}$

$$= V_s \sqrt{\frac{p\delta}{\pi}}.$$

Instantaneous output voltage,

Half wave symmetry $\Rightarrow a_0 = a_n = 0$.

$$b_n = \frac{V_s}{\pi} \left[\int_{\alpha_m}^{\alpha_m + \delta} \cos n\omega t \, d\omega t - \int_{\pi + \alpha_m}^{\pi + \alpha_m + \delta} \cos n\omega t \, d\omega t \right].$$

$$= \frac{V_s}{\pi} \left[\left(\frac{\sin n\omega t}{n} \right)_{\alpha_m}^{\alpha_m + \delta} - \left(\frac{\sin n\omega t}{n} \right)_{\pi + \alpha_m}^{\pi + \alpha_m + \delta} \right].$$

$$= \frac{V_s}{n\pi}\left[\sin(\alpha m+\delta) - \sin n\alpha m - \sin(\pi+\alpha m+\delta) + \sin n(\pi+\alpha m)\right].$$

$$\cancel{\frac{V_s}{n\pi}}$$

For a two pulse,

$$A_n = \frac{2}{\pi}\int_0^{\pi} V_s \sin n\omega t \, d\omega t.$$

$$= \frac{2}{\pi}\int_{\gamma-d/2}^{\gamma+d/2} V_s \sin n\omega t \, d\omega t \times 2.$$

$$= \frac{4V_s}{n\pi}\left(\cos n\omega t\right)_{\gamma+d/2}^{\gamma-d/2}$$

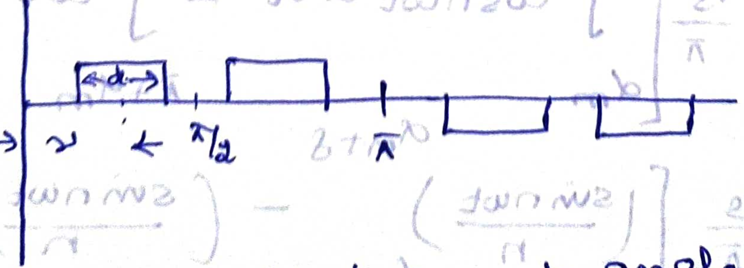
$$= \frac{4V_s}{n\pi}\left[\cos n(\gamma-d/2) - \cos n(\gamma+d/2)\right].$$

$$= \frac{4V_s}{n\pi}\left[\cos n\gamma \cos\frac{nd}{2} + \sin n\gamma \sin\frac{nd}{2} - \cos n\gamma \cos\frac{nd}{2} + \sin n\gamma \sin\frac{nd}{2}\right].$$

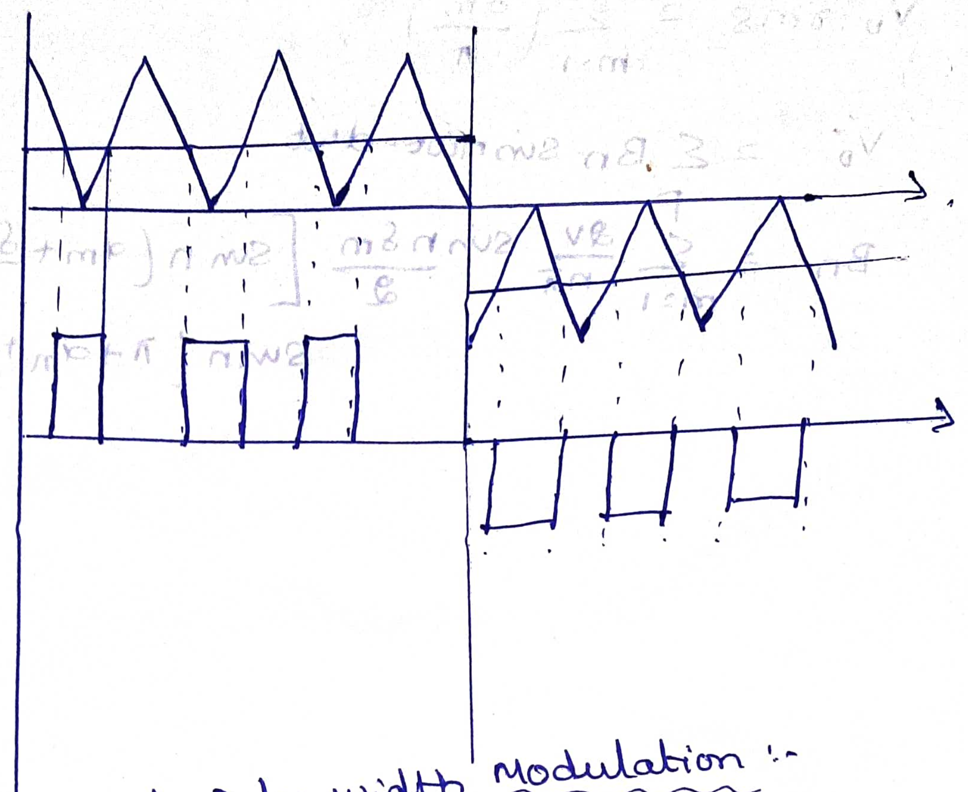
$$V_0 = \sum \frac{8V_s}{n\pi} \sin n\gamma \sin\frac{nd}{2} \sin n\omega t. \quad (n=1,3,5\ldots$$

$$\cancel{b_n = \frac{4}{\pi}}$$

$\gamma = \pi/n$, $d = \frac{2\pi}{n}$, n^{th} harmonic eliminated.



γ – displacement angle.



Sinusoidal Pulse width Modulation :-

⇒ width of pulses are varied in proportion to the amplitude of sine wave.

$$A_n = \frac{2}{\pi}\int_{\alpha_m-\delta/2}^{\alpha_m+\delta/2} V_s \cdot \sin\omega t \cdot d\omega t.$$

$$= \sum \frac{4E_{dc}}{n\pi} \sin\frac{n\delta}{2} \sin n\alpha_m.$$

$$B_n = \frac{2}{\pi}\int_{\alpha_m-\delta/2}^{\alpha_m+\delta/2} \cos n\omega t \cdot d\omega t.$$

$$B_n = \sum_{\alpha_n-\delta/2}^{\alpha_m+\delta/2} \frac{4E_{dc}}{n\pi} \cos\frac{n\delta}{2} \sin n\alpha_m.$$

If the amplitude of sine wave ↓, output voltage ↓

$$M = \frac{A_r}{A_c}.$$

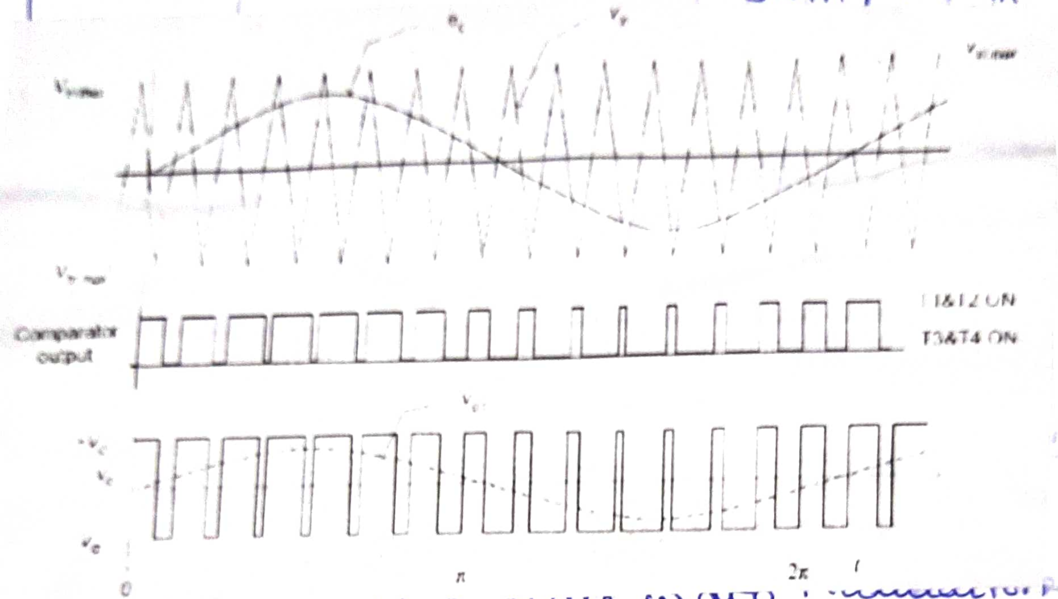
$$\text{Frequency Modulation} = \frac{\omega_c}{\omega_r} = \frac{f_c}{f_r}.$$

step I ; 0-60°, 6, 1 closed

$$V_{0}.rms = \sum_{m=1}^{p} \left(\frac{\delta m}{\pi}\right)^{1/2}.$$

$$V_0 = \sum B_n \sin n\omega t \cdot d\omega t.$$

$$B_n = \sum_{m=1}^{p} \frac{2V}{n\pi} \sin \frac{n \delta m}{2} \left[\sin n\left(\alpha m + \frac{\delta m}{2}\right) - \sin n\left(\pi + \alpha_m + \frac{\delta m}{2}\right)\right].$$



Modified sinusoidal pulse width modulation

=) Near the peak of sine wave, the pulse width donot change, with variation in modulation index.

=) The carrier wave is applied only during (0-60° & 120° to 180°).

Advantages :-

i) Fundamental component increased.

ii) Harmonics reduced.

iii) Switching losses are reduced.

iv) Reduced number of switching devices.

① compare CSI and VSI

CSI	VSI
① Most commonly used for synchronous motor control	Most commonly used for Induction Motor cont
② Peak current rating is smaller	Peak current rating higher.
③ Response time is less	good response time

② what is duty cycle ?

duty cycle can be produced by the comparison of d.c reference signal with the carrier signal.

V_{ref} → amplitude of dc reference signal

V_c → amplitude of carrier signal.

$$\boxed{\delta = \frac{V_{ref}}{V_c}}$$

The ratio of the reference signal to carrier signal gives the modulation index.

③ why thyristors are not preferred for Inverter ?

Thyristors require extra commutation circuit for turn off which result in increased complexity of the circuit. For this reason thyristors are not preferred for inverters.

④ what is a matrix converter ?.

Converters built on the bi-directional, bipolar switches are called matrix converters. They provide a direct power flowing between n-phase ac source and m-phase load.

(5) what are the disadvantages of the harmonics present in the inverter system?

i) The output voltage and waveform becomes distorted one.

ii) Switching losses is increased.

(6) what are the possible methods to control a.c output voltage.

i) AC Voltage control.

ii) Series - inverter control.

UNIT-IV
INVERTERS

1. Why diodes should be connected in antiparallel with the thyristors in inverter circuits?

For RL loads, load current will not be in phase with load voltage and the diodes connected in anti parallel will allow the current to flow when the main thyristors are turned off. These diodes are called feedback diodes.

2. What types of inverters require feedback diodes?

VSI with RL load

3. What is meant a series inverter?

An inverter in which the commutating elements are connected in series with the load is called a series inverter.

4. What is the condition to be satisfied in the selection of L and C in a series inverter?

$R^2 < 4L$

5. What is meant a parallel inverter?

An inverter in which the commutating elements are connected in parallel with the load is called a parallel inverter.

6. What are the applications of a series inverter?

The thyristorised series inverter produces an approximately sinusoidal waveform at a high output frequency, ranging from 200 Hz to 100kHz. It is commonly used for fixed output applications such as a. Ultrasonic generator. b. Induction heating. c. Sonar Transmitter d. Fluorescent lighting.

7. How is the inverter circuit classified based on commutation circuitry?

a. Line commutated inverters. b. Load commutated inverters. c. Self commutated inverters. d. Forced commutated inverters.

8. What is meant by McMurray inverter?

It is an impulse commutated inverter which relies on LC circuit and an auxiliary thyristor for commutation in the load circuit.

9. What are the applications of a CSI?

a. Induction heating b. Lagging VAR compensation c. Speed control of ac motors d. Synchronous motor starting.

10. What is meant by PWM control?

In this method, a fixed dc input voltage is given to the inverter and a controlled ac output voltage is obtained by adjusting the on and off periods of the inverter components. This is the most popular method of controlling the output voltage and this method is termed as PWM control .

11. What are the advantages of PWM control?

a. The output voltage can be obtained without any additional components.
b. Lower order harmonics can be eliminated or minimized along with its output voltage control. As the higher order harmonics can be filtered easily, the filtering requirements are minimized.

12. What are the disadvantages of the harmonics present in the inverter system?

a. Harmonic currents will lead to excessive heating in the induction motors. This will reduce the load carrying capacity of the motor.
b. If the control and the regulating circuits are not properly shielded, harmonics from power ride can affect their operation and malfunctioning can result.
c. Harmonic currents cause losses in the ac system and can even some time produce resonance in the system. Under resonant conditions, the instrumentation and metering can be affected.
d. On critical loads, torque pulsation produced by the harmonic current can be useful.

13. What are the methods of reduction of harmonic content?

a. Transformer connections b. Sinusoidal PWM c. Multiple commutation in each cycle d. Stepped wave inverters

15. What are the disadvantages of PWM control?

SCRs are expensive as they must possess low turn-on and turn-off times.

16. What does ac voltage controller mean?

It is device which converts fixed alternating voltage into a variable voltage without change in frequency.

17. What are the applications of ac voltage controllers?

a. Domestic and industrial heating b. Lighting control c. Speed control of single phase and three phase ac motors d. Transformer tap changing

18. What are the advantages of ac voltage controllers?

a. High efficiency b. Flexibility in control c. Less maintenance

19. What are the disadvantages of ac voltage controllers?

The main draw back is the introduction of harmonics in the supply current and the load voltage waveforms particularly at low output voltages .

20. What are the two methods of control in ac voltage controllers?

a. ON-OFF control b. Phase control

UNIT - IV

7) Why the THD has to be mitigated ?

1. To improve power factor and reduce system loss.
2. Minimise interference with other equipment.
3. To improve system voltage/current waveform.
4. To prevent nuisance tripping of fuse and circuit Breakers.

8. What are the purposes of feed back diodes in Inverter:

For inductive load, current i_o will not be in phase with voltage V_o and diodes connected in antiparallel with thyristors will allow the current to flow when the main thyristors are turned off. These diodes are called feedback diodes.

9. Mention the PWM methods in Inverters.

i) Single pulse modulation.
ii) Multiple pulse modulation.
iii) Sinusoidal pulse modulation.
iv) Modified sinusoidal pulse width modulation.

10. What are the advantages and disadvantages of resonant pulse converter:

Advantages :-

i) Switching losses are less.
ii) Less electromagnetic Interference.
iii) Operating switching frequency is high.
iv) Efficiency is high.

Disadvantages :

1) Limited frequency.
2) Larger size.
3) Heavy weight.
4) Power dissipation may occur in any working condition.

Study of Switching Devices - Diode :

Power Diode :-

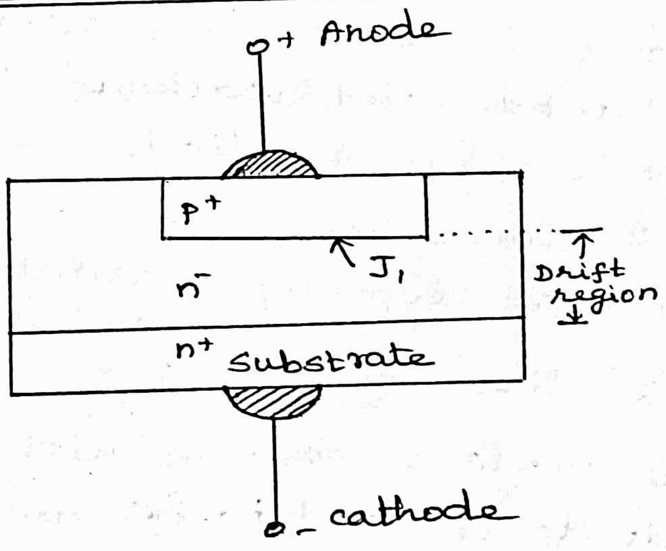
A low power diode called signal diode, is a pn-junction device. A high power diode, called Power diode is also a pn-junction device but with constructional features somewhat different from a signal diode.

The voltage, current and power ratings of Power diodes and transistors are much higher than the corresponding ratings for signal devices. Power devices operate at lower switching speeds whereas signal diodes and transistors operate at higher switching speeds.

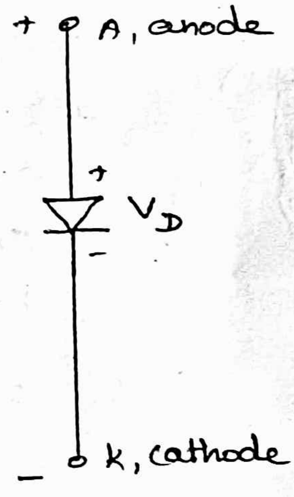
BASIC STRUCTURE OF POWER DIODES :

doping density

P	N	Junction
10^{16} to $10^{17} cm^{-3}$	same	pn
$10^{19} cm^{-3}$ to	$10^{17} cm^{-3}$	p^+n
$10^{19} cm^{-3}$	10^{13}	p^+n^-
$10^{19} cm^{-3}$	$10^{19} cm^{-3}$	p^+n^+



(a) Structural features of Power diode



circuit symbol.

It consists of heavily doped n^+ substrate. On this substrate, a lightly doped n^- layer is epitaxially grown. Now a heavily doped p^+ layer is diffused into n^- layer to form the anode of power diode.

n^- layer is the basic structural feature not found in signal diodes.

The function of n^- layer is to absorb the depletion layer of the reverse biased p^+n^- junction J_1. The break down voltage needed in a power diode governs the thickness of n^- layer. Greater the breakdown voltage, more the n^- layer thickness.

The drawback of n^- layer is to add significant ohmic resistance to the diode when it is conducting a forward current. This leads to large power dissipation in the diode.

CHARACTERISTICS OF POWER DIODES :

Power diode is a two-terminal, p-n semiconductor device. The two terminals of diode are called anode and cathode. Two important characteristics of power diodes are

(1) Diode V-I characteristics.

(2) Diode Reverse Recovery characteristics.

(1) Diode V-I characteristics

When anode is positive with respect to cathode, diode is said to be forward biased. With increase of the source voltage V_s from zero value, initially diode current is zero. From $V_s = 0$ to cut in voltage, the forward diode current is very small. Cut in voltage is also known as threshold voltage or turn on voltage. Beyond cut in voltage, the diode current rises rapidly and the diode is said to conduct.

T = absolute temperature in kelvin ($K = 273 + c^{\circ}$).

K = Boltzmann's constant ; 1.3806×10^{-23} J/K.

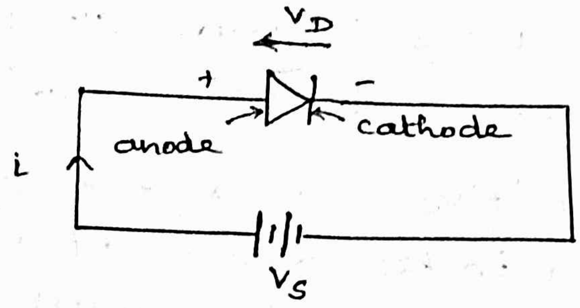
When cathode is positive with respect to anode, the diode is said to be reverse biased. In the reverse biased condition, a small reverse current called leakage current of the order of microamperes or milliamperes flows.

At reverse breakdown, voltage remains const, but reverse current becomes quite high limited only by the external circuit Resistance. A large reverse breakdown voltage, associated with high reverse current leads to excessive power loss that may destroy the diode.

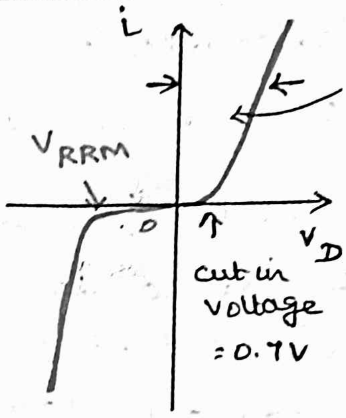
For an ideal diode, $V_D = 0$, reverse leakage current $= 0$, cut in voltage $= 0$, reverse breakdown voltage V_{RRM} is infinite.

Peak inverse voltage (PIV) is the largest reverse voltage to which a diode may be subjected during its working. PIV is the same as V_{RRM}.

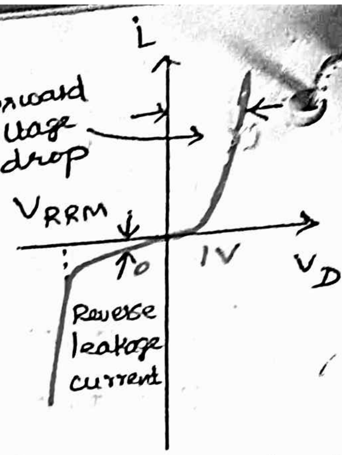
The power diodes are now available with forward current ratings of 1A to several thousands amperes and with reverse voltage ratings of 50V to 5000 V or more.



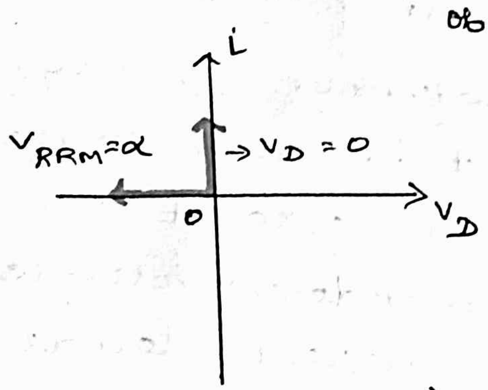
(a) Forward biased Power diode



b) i-V characteristics of signal diode



c) i-v characteristics of power diode.



d) iv characteristics of ideal diode

For silicon diode, the cut in voltage is around 0.7V. when diode conducts, there is a forward voltage drop of the order of 0.8 to 1V.

The characteristics can be expressed by an equation known as Schockley diode Equation, and is given by

$$I_D = I_S\left[e^{V_D/nV_T} - 1\right]$$

where I_D = current through the diode A

V_D = Diode voltage with anode positive with respect to cathode (V).

I_S = leakage current range : 10^{-6} to 10^{-15} A.

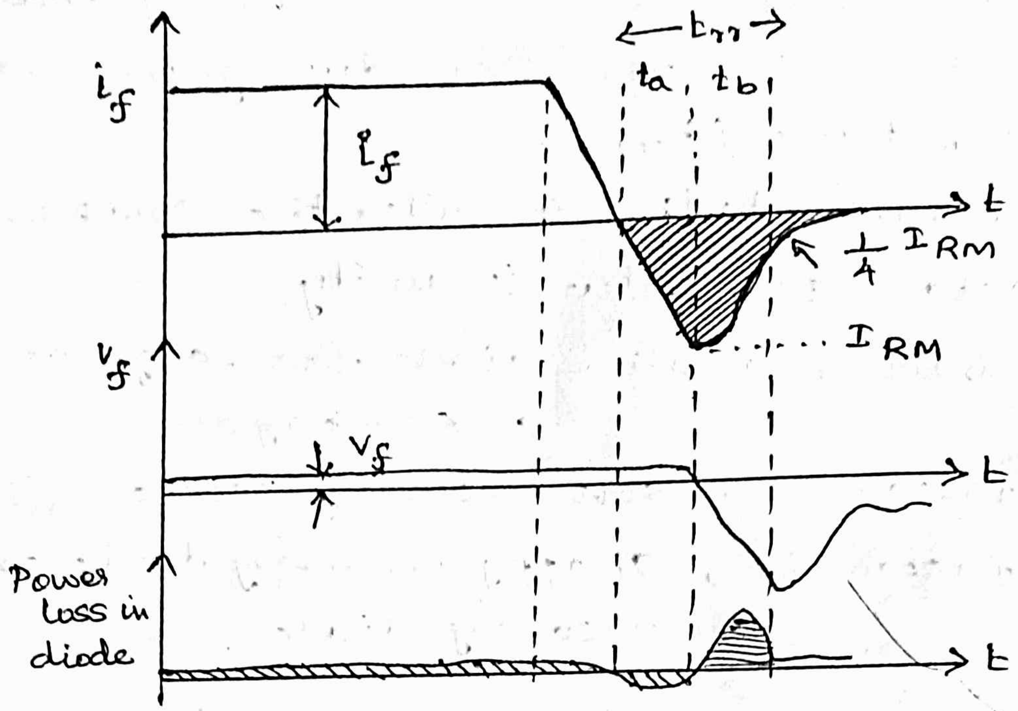
n = emission co-efficient, value : 1 to 2.

V_T = Thermal voltage, it is given by

$$V_T = \frac{KT}{Q}$$

where Q = electron charge 1.6022×10^{-19} C).

DIODE REVERSE RECOVERY CHARACTERISTICS :-



⇒ After the forward diode current decays to zero, the diode continues to conduct in the reverse direction because of the pressure of stored charges in the depletion region.

⇒ Reverse recovery time t_{rr} is defined as the time between the instant forward diode current becomes zero, and the instant reverse recovery current decays to 25% of its reverse peak value I_{RM}.

⇒ $t_{rr} = t_a + t_b$

Time t_a = Time between zero crossing of forward current and peak reverse current I_{RM}. ∴ charge stored in depletion layer is removed.

Time t_b = Measured from the instant of reverse peak value I_{RM} to the instant when $0.25\, I_{RM}$ is reached. Charge from the semiconductor layer is removed.

⇒ The shaded area in fig (a) represents the stored charge or reverse recovery charge Q_R which must be removed during the reverse recovery time (t_{rr}).

⇒ The ratio t_b/t_a is called the softness factor (or) S-factor. Its value is unity.

S factor small - diode has large oscillatory over voltage

S factor = 1, soft recovery Diode

S factor < 1, snappy recovery diode or fast recovery diode.

Peak inverse current I_{RM} can be expressed as

$$I_{RM} = t_a \cdot \frac{di}{dt} \quad \ldots\ldots\ldots (1)$$

$\frac{di}{dt}$ = rate of change of reverse current.

$$Q_R = \frac{1}{2} I_{RM} \times t_{rr}$$

$$I_{RM} = \frac{2Q_R}{t_{rr}} \quad \ldots\ldots\ldots (2)$$

If $t_{rr} \equiv t_a$, from eqn ①

$$I_{RM} = t_{rr} \cdot \frac{di}{dt} \quad \ldots\ldots (3).$$

From eqn ② & ③, we get,

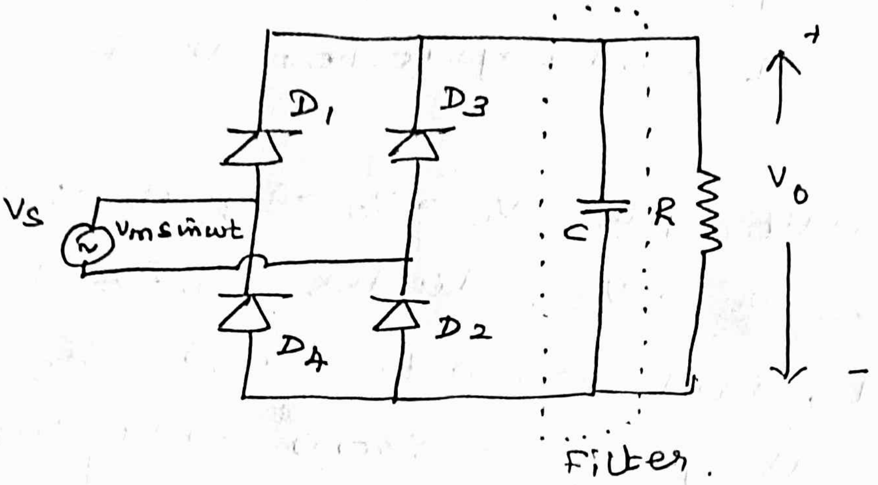
$$t_{rr} \cdot \frac{di}{dt} = \frac{2Q_R}{t_{rr}}$$

$$t_{rr} = \left[\frac{2Q_R}{di/dt}\right]^{1/2}$$

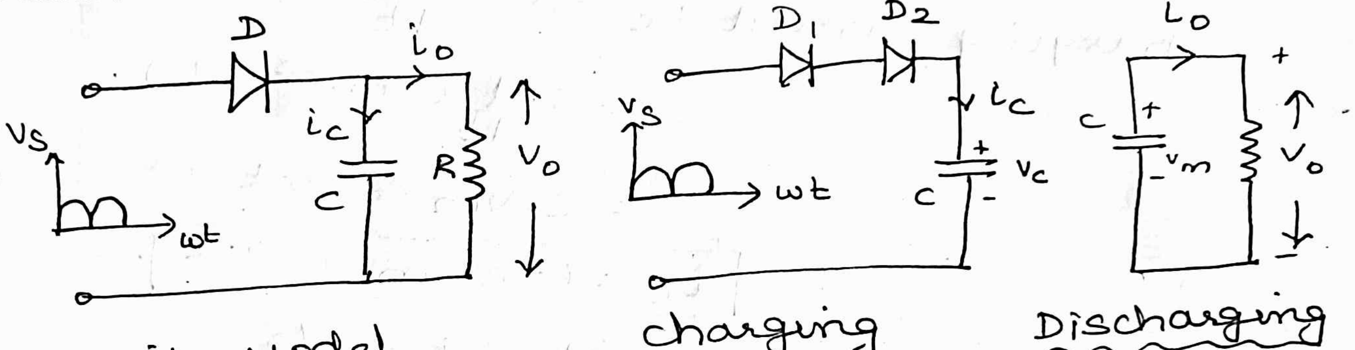
from eqn ③, $I_{RM} = \left[\frac{2Q_R}{di/dt}\right]^{1/2} \cdot di/dt$

$$\boxed{I_{RM} = \left[2 Q_R \frac{di}{dt}\right]^{1/2}}$$

capacitor Filter (C-Filter).



$\Rightarrow$ A capacitor C directly connected across the load serves to smoothen out the dc output wave.



circuit Model

$\Rightarrow$ From $\omega t = 0$ to $\omega t = \theta$, source voltage V_s is less than capacitor voltage $V_c = V_o$. Therefore diodes D_1, D_2 are reverse biased and cannot conduct.

$\Rightarrow$ After $\omega t = \theta$, source voltage V_s exceeds V_o ($= V_c$), diodes D_1, D_2 get forward biased and begin to conduct. Source voltages charges capacitor from V_2 to V_m at $\omega t = \pi/2$.

area in fig (a) represents the need

After $\omega t = \pi/2$, source voltage begins to decrease faster than the capacitor voltage. Diodes D_1, D_2 are reverse biased and capacitor discharges through R.

=> In the next half cycle, $V_c = V_o = V_2$ at $\omega t = \pi + \theta$.
After $\omega t = \pi + \theta$, $V_s > V_c$, diodes D_3, D_4 get forward biased, and begin to conduct.
The capacitor voltage rises from V_2 to V_m at $\omega t = 3\pi/2$

Charging of capacitor :

Charging current $i_c = C \cdot \frac{dV_s}{dt}$

$$= C \cdot \frac{d}{dt}(V_m \sin \omega t).$$

$$= C \cdot V_m \cos \omega t \cdot \omega.$$

$$\boxed{i_c = \omega C V_m \cos \omega t}.$$

Energy stored in c at $\omega t = \pi/2$,

$$i_c = \omega C V_m \cos 90^\circ = 0.$$

$$V_c = V_m.$$

Energy stored in C at $\omega t = \pi/2 = \frac{1}{2} C V_m^2$

Discharging of capacitor :-

$$V_o = V_m e^{-t/RC}$$

Peak to peak ripple voltage

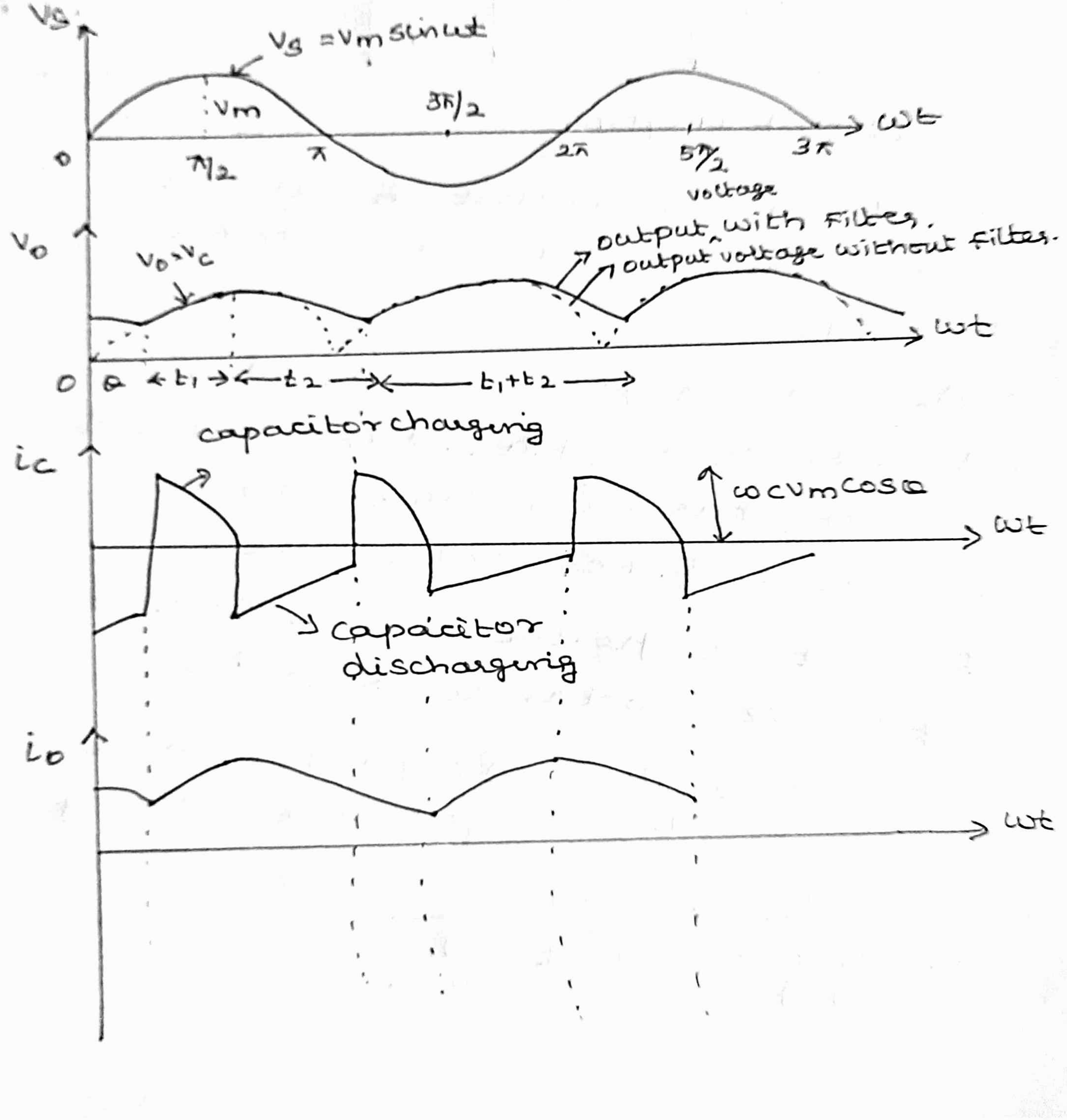
$$V_{rpp} = \frac{V_m t_2}{RC}$$

Peak value of ripple voltage $V_{rp} = \frac{V_{rpp}}{2}$

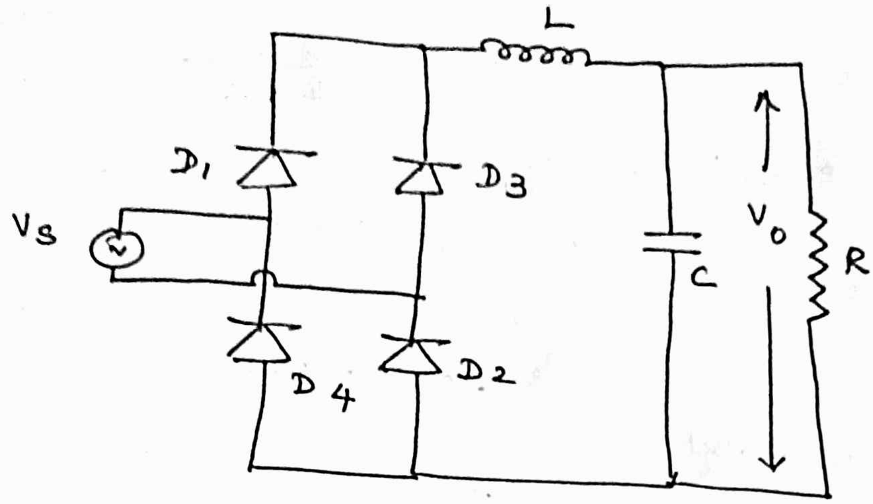
$$= \frac{V_m}{4fRC}$$

Ripple factor RF = $\frac{\text{Ripple voltage } V_r}{\text{Average output voltage, } V_o}$

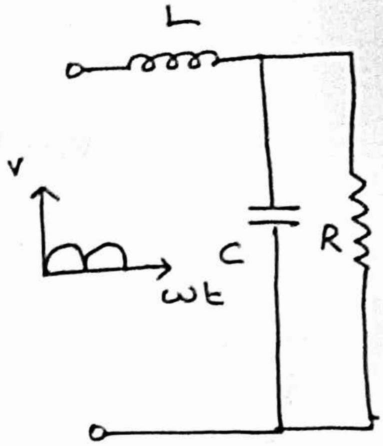
$$= \frac{1}{\sqrt{2}\,[4fRC - 1]}$$



LC Filter.



Circuit Diagram



Equivalent circuit.

=> The L-C filter consists of inductor L in series with the load and capacitor C across the load.

=> The LC filter possesses the advantages of both L filter & C filter. Ripple factor in L-C filter has lower value than that obtained by either L filter (or) C filter.

=> R must be greater than n^{th} harmonic capacitive reactance. $R >> \frac{1}{n\omega C}$.

$$R = \frac{10}{n\omega C}$$ => capacitor provides effective filtering.

$$I_n = \frac{V_n}{n\omega L - \frac{1}{n\omega C}}$$

(nth harmonic current)

n^{th} harmonic component of load voltage V_{on} :

$$V_{on} = \left[\frac{-1}{(n\omega)^2 LC - 1}\right] . V_n .$$

ripple voltage $V_r = \left[\sum_{n=2,4,6}^{\infty} V_{on}^2\right]^{1/2}$

$$C = \frac{10}{2\omega R}$$

$$VRF = \frac{\sqrt{2}}{3}\left[\frac{1}{(2\omega)^2 LC - 1}\right].$$

Single phase Half-wave Rectifier

Rectification is the process of conversion of alternating input voltage to direct output voltage. A rectifier converts ac power to dc power. The output voltage cannot be controlled.

Types :

1) One pulse
2) Two pulse
3) Three pulse (or) n pulse

Pulse number : number of load current (or voltage) pulses during one cycle of ac source voltage.

Single phase Half wave Rectifier :-

→ Simplest type of uncontrolled rectifier.

(a) R load :-

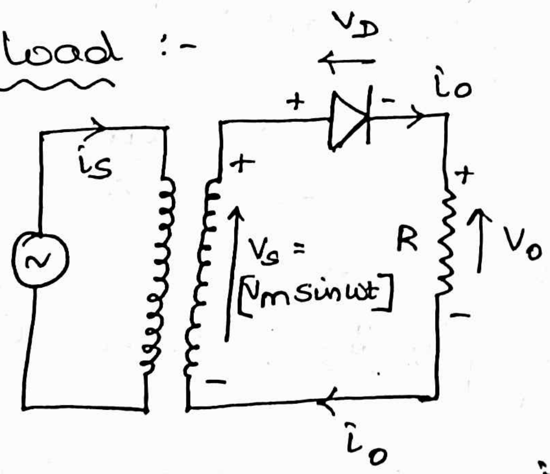
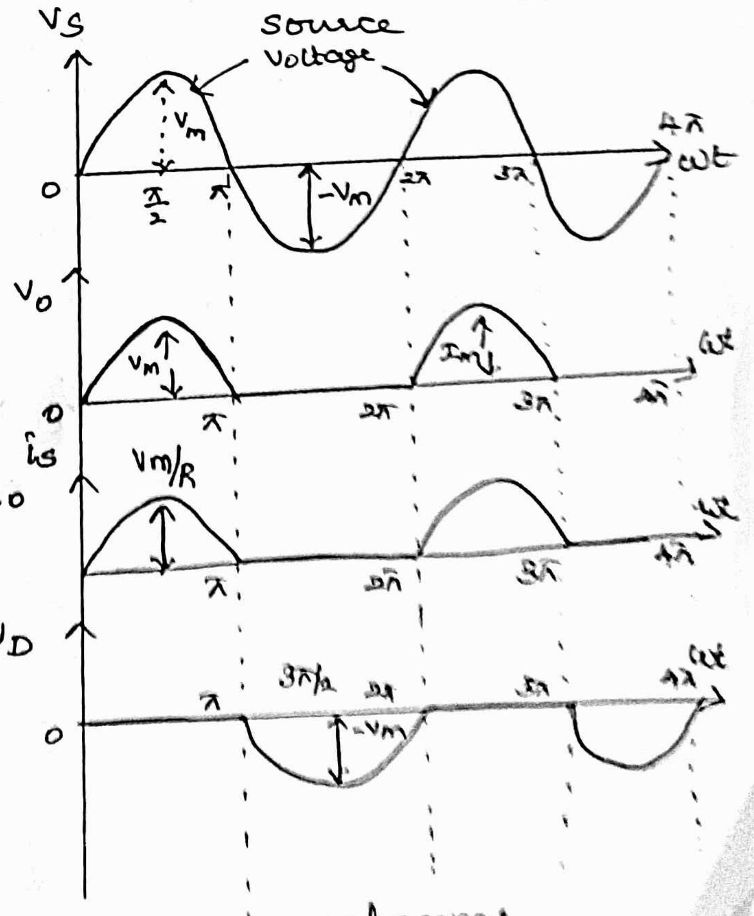


Fig : Single phase half wave diode Rectifier with R-load circuit diagram



Waveforms.

(a) R - Load :

* During the positive half cycle, diode is forward biased, it therefore conducts from $\omega t = 0°$ to $\omega t = \pi$.

$V_o = V_s$.

$i_o = \frac{V_o}{R}$

* At $\omega t = \pi$, $V_o = 0$, $i_o = 0$. V_s tends to become negative after $\omega t = \pi$, diode D is reverse biased. It is therefore turned off and goes into blocking state.

$V_o = 0$, $i_o = 0$ at $\omega t = \pi$ to $\omega t = 2\pi$.

Diode voltage V_D is zero when diode conducts.

Source voltage is sinusoidal $V_s = V_m \sin\omega t$.

Average output or load voltage,

$$V_o = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t . d(\omega t)\right].$$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi} = \frac{V_m}{\pi}.$$

Rms value of output voltage,

$$V_{or} = \left[\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\omega t . d(\omega t)\right]^{1/2}.$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2} d(\omega t)\right]^{1/2}.$$

$$= \frac{V_m}{\sqrt{2\pi}\times\sqrt{2}}\left(\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}\right)^{1/2}.$$

$$= \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\left[\pi - \frac{\sin 2\pi}{2}\right]^{1/2} = \frac{V_m}{\sqrt{2}\sqrt{2}\cancel{\sqrt{\pi}}}\times\cancel{\sqrt{\pi}}$$

$$= \frac{V_m}{2}.$$

Average value of load current, $I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}$.

Rms value of load current $I_{or} = \frac{V_{or}}{R} = \frac{V_m}{2R}$.

Peak value of diode current $= \frac{V_m}{R}$.

→ Peak inverse voltage, PIV is an important parameter in the design of rectifier circuits.

→ PIV is the maximum voltage that appears across the device during its blocking state.

$$PIV = V_m = \sqrt{2} \cdot V_s$$

$= \sqrt{2} \times$ rms value of transformer secondary voltage.

Power delivered to resistive load

(rms load voltage) (rms load current).

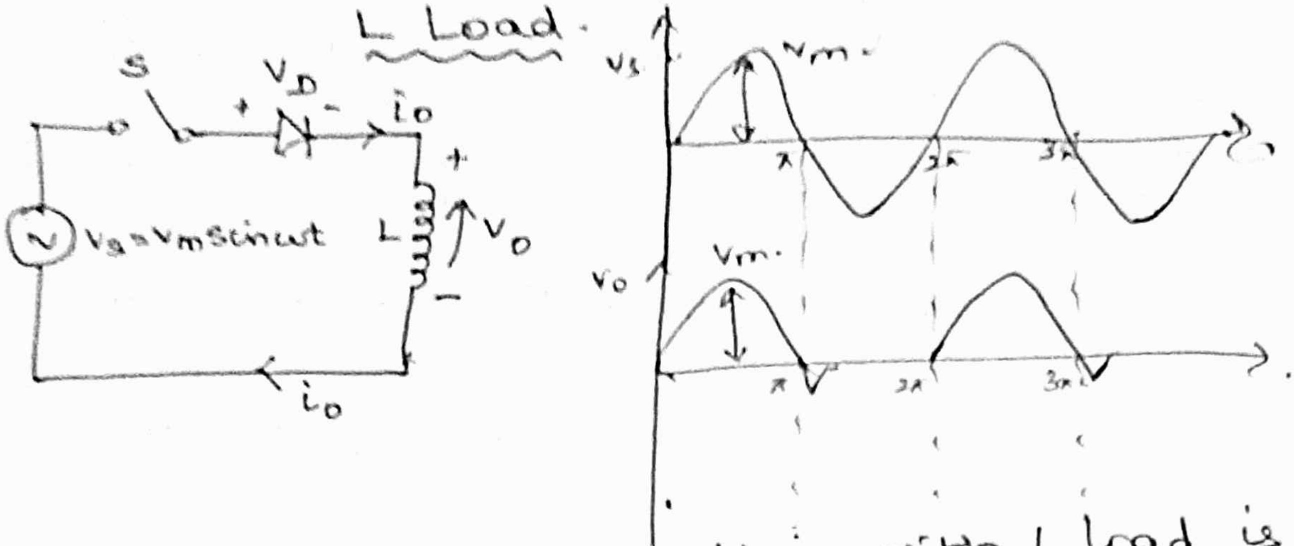
$$= V_{or} I_{or} = \frac{V_m}{2} \times \frac{V_m}{2R} = \frac{V_m^2}{4R} = \frac{V_s^2}{2R}.$$

$$\text{Input power factor} = \frac{\text{Power delivered to load}}{\text{Input VA}}$$

$$= \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = \frac{V_{or}}{V_s} = \frac{\sqrt{2} V_s}{2 V_s}$$

$= 0.707$ lag.

<u>L Load.</u>



⇒ 1ϕ Half wave diode rectifier with L load is shown in fig.

⇒ when switch S is closed at $\omega t = 0$, diode starts conducting.

$$V_s = V_o = L \cdot \frac{di_o}{dt} = V_m \sin \omega t.$$

$$i_o = \frac{V_m}{L} \int \sin \omega t \cdot dt. \quad \text{—— (1).}$$

$$= \frac{-V_m}{\omega L} \cos \omega t + A.$$

At $\omega t = 0$, $i_o = 0$, $\quad 0 = \frac{-V_m}{\omega L} + A.$

$$\boxed{A = \frac{V_m}{\omega L}} \quad \text{—— (2).}$$

Sub eqn (2) in eqn (1),

$$i_o = \frac{-V_m}{\omega L} \cos \omega t + \frac{V_m}{\omega L}.$$

$$= \frac{V_m}{\omega L}\left[1 - \cos \omega t\right]. \quad \text{—— (3).}$$

$$V_o = L \cdot \frac{di_o}{dt} = L \frac{V_m}{\omega L}\left[\sin \omega t\right] \cdot \omega.$$

$$= V_m \sin \omega t \Rightarrow V_s.$$

Peak value of current I_{max} occurs at $\omega t = \pi$.

$$I_{max} = \frac{V_m}{\omega L}(1+1) = \frac{2V_m}{\omega L} \quad \text{---} \quad (4).$$

(From eqn (3) => $\frac{V_m}{\omega L}\left[1-\cos\omega t\right] = i_o$).

sub $i_o = I_{max}$, $\omega t = \pi$).

Average value of current $I_o = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m}{\omega L}(1-\cos\omega t)\, d(\omega t)$

$$= \frac{V_m}{\omega L} = \frac{I_{max}}{2} \quad \text{---} \quad (5).$$

Rms value of fundamental current I_{1r} is given by,

$$I_{1r} = \left[\frac{1}{2\pi}\left(\frac{V_m}{\omega L}\right)^2 \int_0^{2\pi} (\cos\omega t)^2\, d(\omega t)\right]^{1/2}.$$

$$= \frac{V_m}{\sqrt{2}\cdot\omega L} = \frac{\sqrt{2}V_s}{\sqrt{2}\omega L} = \frac{V_s}{\omega L} = \frac{I_o}{\sqrt{2}}.$$

Rms value of rectified current $= \left[I_o^2 + I_{1r}^2\right]^{1/2}$.

$$= \left[I_o^2 + \frac{I_o^2}{2}\right]^{1/2}$$

$$= 1.225\, I_o.$$

Voltage across diode $V_D = 0$.

① A 1ϕ 230 V, 1 kW heater is connected across 1ϕ 230V, 50 Hz supply through a diode. Calculate the power delivered to the heater element. Find also the peak diode current and input power factor.

$P = VI$.
$P = V \times \frac{V}{R}$.
$\boxed{P = \frac{V^2}{R}}$

Heater Resistance $R = \frac{V^2}{P}$

$= \frac{230^2}{1000}$

Rms value of output voltage $= \frac{V_m}{2}$

$V_m = \sqrt{2} \times V_s$.

$= \frac{\sqrt{2} \times V_s}{2} = \frac{\sqrt{2} \times 230}{2}$

Power absorbed by heater element

$P = \frac{V_{or}^2}{R} = \frac{2 \times 230^2}{4} \times \frac{1000}{230^2}$

$= \frac{1000}{2}$

$= 500W$.

Peak value of diode current $= \frac{V_m}{R}$.

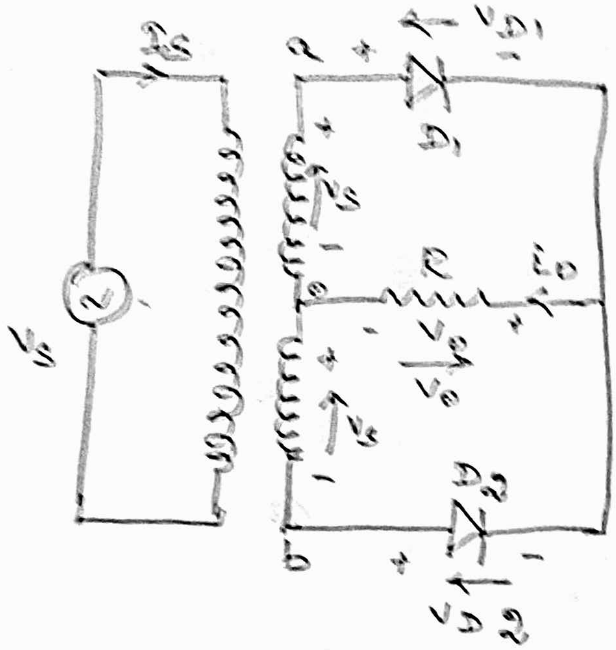
$= \frac{\sqrt{2} \times 230}{230^2} \times 1000$

$= 6.1478$ A.

Input power factor $= \frac{V_{or}}{V_s} = \frac{\sqrt{2} \times 230}{2} \times \frac{1}{230}$

$= 0.707$ lag.

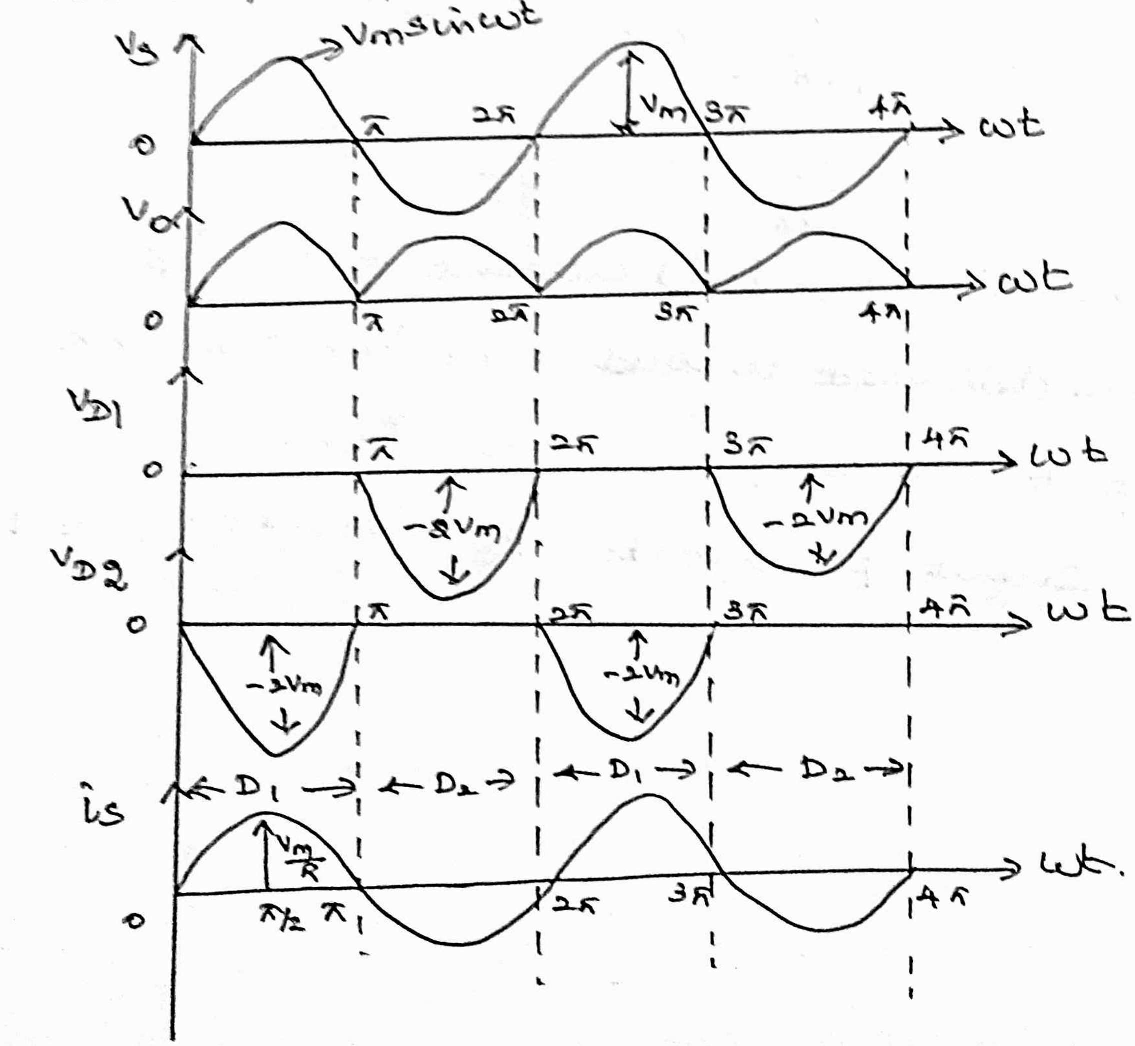
Single phase Full wave midpoint diode Rectifier



→ The turns ratio from each secondary to primary is taken as unity.

→ when a is +ve with respect to b (or) mid point o diode D_1 conducts for π radians. D_2 reverse biased and voltage of $2V_s$.

→ In the next half cycle, b is +ve with respect to a or mid point o and Diode D_2 conducts. Diode D_1 experiences a reverse voltage of $2V_s$.



Average output voltage $V_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d(\omega t)$.

$$V_0 = \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi} = \frac{V_m}{\pi}\left[-\cos(\pi) + \cos 0\right].$$

$$V_0 = \frac{2V_m}{\pi}$$

Average output current, $I_0 = \frac{V_0}{R} = \frac{2V_m}{\pi R}$.

Rms value of output voltage

$$V_{or} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\omega t \, d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\pi}}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left\{\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}\right\}^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\pi - \frac{\sin 2\pi}{2} - 0\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}} \times \sqrt{\pi} = \frac{V_m}{\sqrt{2}} = V_s.$$

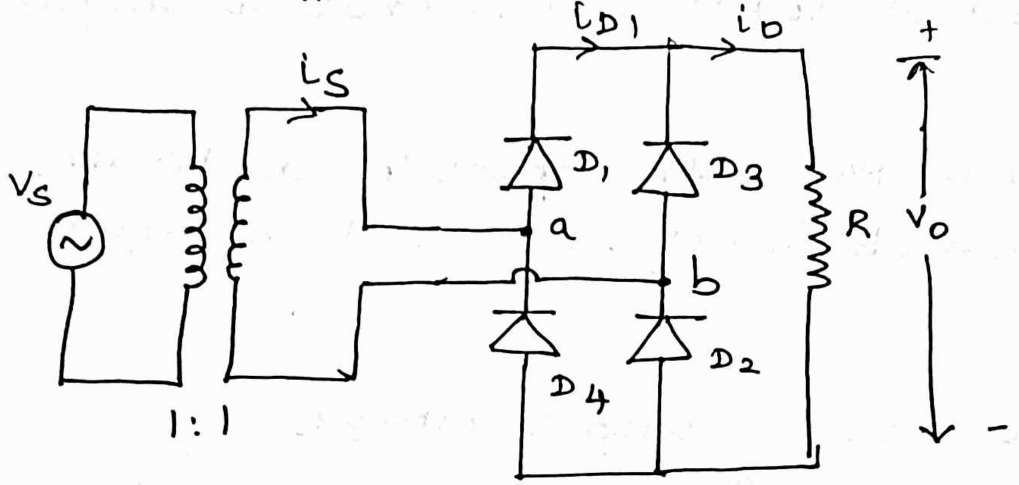
Rms value of load current $I_{or} = \frac{V_s}{R}$

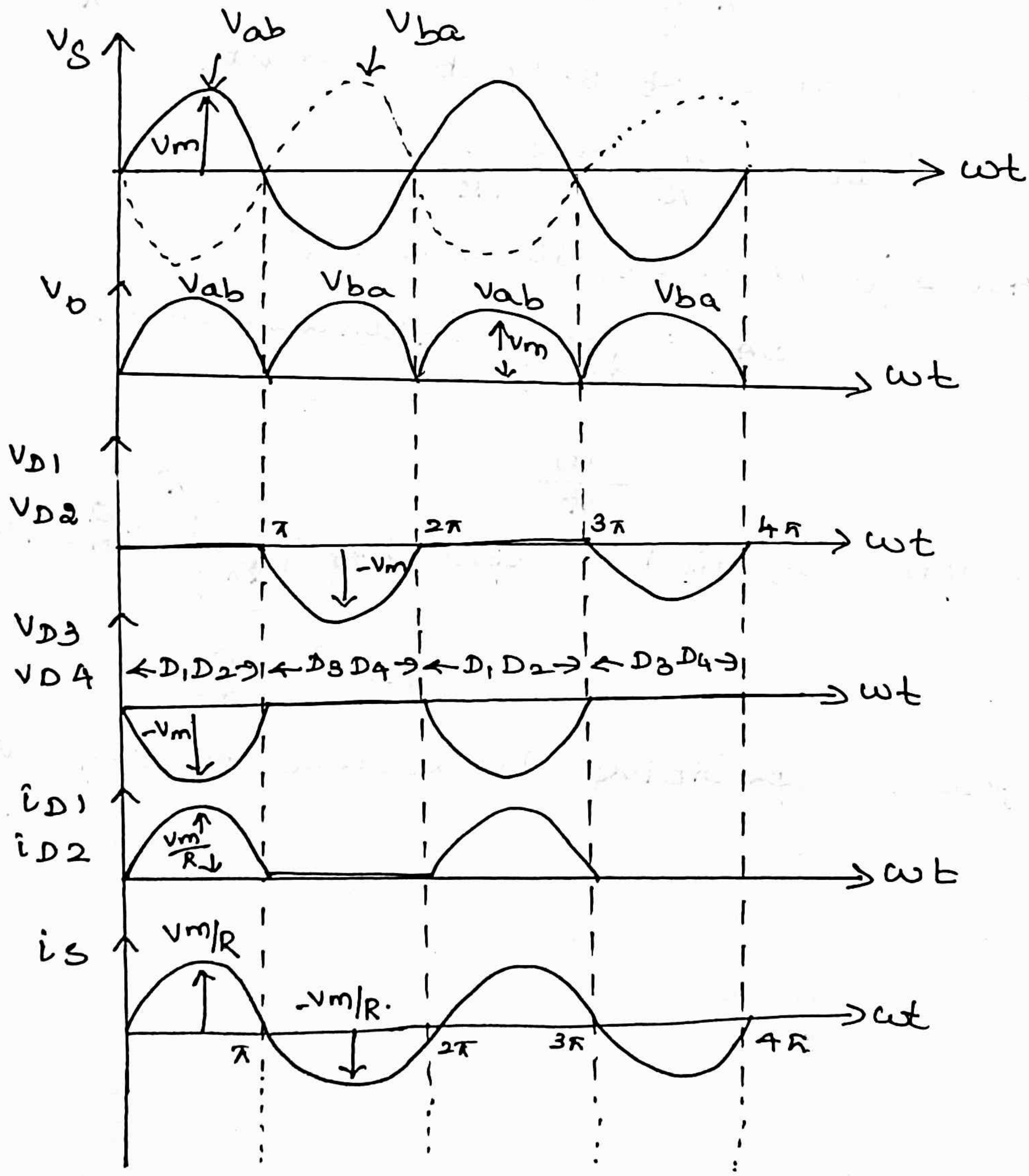
Power delivered to load $= V_{or} \cdot I_{or} = I_{or}^2 \times R$

Input volt-ampere $= V_s \cdot I_{or}$.

Input powerfactor $P_f = \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = 1$.

Single phase Full wave diode Bridge Rectifier :





⇒ When a is +ve with respect to b diodes, D_1, D_2 Conduct together and that of output voltage is V_{ab}.

⇒ When b is +ve with respect to a diodes, D_3, D_4 Conduct together and that of output voltage is V_{ba}.

Average value of output voltage.

$$V_o = \frac{1}{\pi}\int_0^{\pi} V_m \cdot \sin \omega t \cdot d(\omega t) = \frac{2V_m}{\pi}$$

Average value of output current,

$$I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R}$$

Average value of diode current

$$I_{DA} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t \cdot d(\omega t).$$

$$= \frac{I_m}{\pi}$$

RMS Value of diode current $I_{Dr} = \left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t \cdot d(\omega t)\right]^{1/2}$

$$I_{Dr} = \frac{I_m}{2}$$

Peak repetitive diode current $I_m = \frac{V_m}{R}$.

Input Performance Parameters.

(1) Input power-factor :

The input power factor is defined as the ratio of mean input power (real power) to the total rms input voltamperes (Apparent power) given to the converter system.

$$\text{Power Factor} = \frac{\text{Real power } V_s \times I_{s1} \cdot \cos\phi_1}{\text{Apparent power } V_s \cdot I_s}$$

$$= \frac{I_{s1}}{I_s} \cdot \cos\phi_1$$

(2) Input displacement Factor (DF) :

The phase angle between sinusoidal supply voltage V_s & Fundamental component I_{s1} of supply current I_s is ϕ_1.

$\phi_1 \rightarrow$ displacement Angle.

Its cosine is called input displacement factor.

$$DF = \cos\phi_1.$$

(3) Input current Distortion Factor (CDF)

It is defined as the ratio of the rms value of fundamental component I_{s1} of the input current to the rms value of input I_s.

$$CDF = \frac{I_{s1}}{I_s}$$

; Power factor = Input current distortion factor × Input displacement Factor.

(4) Input current Harmonic factor (HF):

The Harmonic Factor (HF) is equal to the rms value of all the harmonics divided by the rms value of fundamental component of the input current.

$$I_h = \sqrt{I_s^2 - I_{s1}^2}$$

$$HF = \frac{I_h}{I_{s1}} = \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = \frac{\sum_{n=2}^{\infty} I_{sn}}{I_{s1}}$$

I_{sn} ⇒ rms value of nth Harmonic content.

(5) Crest Factor (CF)

Crest factor for input current is defined as the ratio of peak input current I_{sp} to its rms value I_s.

$$CF = \frac{I_{sp}}{I_s}.$$

OUTPUT PERFORMANCE PARAMETERS

The load or output voltage and the load or output current at the output terminals of ac to dc converters are unidirectional but pulsating in nature. Fourier series is used to express these output quantities in terms of two components (i) Average or dc value.

(ii) AC component superimposed on dc value.

In general, average value of output quantity y is,

$$Y_o = Y_{dc} = \frac{1}{T}\int_{t_1}^{t_1+T} y \cdot dt.$$

$$Y_{or} = \left[\frac{1}{T}\int_{t_1}^{t_1+T} y^2 dt\right]^{1/2}$$

y → Instantaneous value of the function in terms of t.

T → Time period for one cycle of y variation.

output dc power P_{dc} = average output voltage V_o × average output current I_o.

$$P_{dc} = V_o \times I_o.$$

~~output~~ Input ac power $P_{ac} = V_{or} \times I_{or}$.

(i) Rectification ratio (η) :

Rectification ratio also called efficiency of a converter is defined as the ratio of dc output power P_{dc} to ac input power P_{ac}.

$$\eta = \frac{P_{dc}}{P_{ac}}.$$

Ripple value of the ac component of output voltage, $V_r = \sqrt{V_{or}^2 - V_o^2}$

V_r → Ripple voltage

(ii) Form Factor (FF)

It is defined as the ratio of rms value of output voltage V_{or} to the dc value V_o of output voltage.

$$FF = \frac{V_{or}}{V_o}$$

(3) Voltage ripple factor (VRF) :

It is defined as the ratio of ripple voltage V_r to the average output voltage V_o.

$$VRF = \frac{V_r}{V_o}$$

$$VRF = \left[\left(\frac{V_{or}}{V_o}\right)^2 - 1\right]^{1/2} = \sqrt{FF^2 - 1}$$

$$FF = \sqrt{VRF^2 + 1}$$

(4) Per unit average output voltage :

It is defined as the ratio of the average output voltage V_o to the average output voltage V_{om}.

$$\boxed{V_{o.pu} = \frac{V_o}{V_{om}}}$$

(5) current ripple factor (CRF) :

It is defined as the ratio of rms value of all harmonic components of output current to the dc component I_o of the output current.

$$CRF = \frac{I_r}{I_o} = \frac{\sqrt{I_{or}^2 - I_o^2}}{I_o} = \left[\left(\frac{I_{or}}{I_o}\right)^2 - 1\right]^{1/2}.$$

$I_{or} \to$ rms value of output current including dc & harmonics.

$I_r \to$ rms value of all harmonic components of output current.

$I_o \to$ dc component of output current.

$$\boxed{I_{or}^2 = I_o^2 + I_r^2}.$$

(6) Transformer utilization Factor (TUF) :

$$TUF = \frac{P_{dc}}{V_s I_s}$$

$$TUF = \frac{P_{dc}}{\text{Transformer VA rating}}$$

$$\text{Transformer VA rating} = \frac{P_{dc}}{TUF}.$$

Rectifier produces a perfect dc output voltage

(i) rms value = dc value

(ii) FF = 1.

(iii) ac component of output voltage = 0

iv) HF = 0.

v) PF = 1.0.

vi) TUF = 1.

Comparison of single phase Diode Rectifiers

S.No	Parameters	Half wave (one pulse)	Full wave (Two pulse) Centre tap (M-2)	Full wave (Two pulse) Bridge (B-2)
1.	DC output voltage, V_0	V_m/π	$2V_m/\pi$	$2V_m/\pi$
2.	rms value of output voltage, V_{or}	$V_m/2$	$V_m/\sqrt{2}$	$V_m/\sqrt{2}$
3.	Ripple voltage, V_r	$\sqrt{V_{or}^2 - V_0^2} = 0.3856V_m$	$\sqrt{V_{or}^2 - V_0^2} = 0.3077V_m$	$\sqrt{V_{or}^2 - V_0^2} = 0.3077V_m$
4.	Voltage Ripple Factor (VRF)	$\frac{V_r}{V_0} = 1.211$	0.483 $0.3077V_m \times \frac{\pi}{2V_m} = 0.483$	$0.3077V_m \times \frac{\pi}{2V_m} = 0.483$
5.	Rectification efficiency, η	$\eta = \frac{P_{dc}}{P_{ac}} = 0.4053$	0.8106	0.8106
6.	Transformer utilization factor (TUF)	$TUF = \frac{P_{dc}}{V_s I_s} = 0.2865$	0.672	0.8106
7.	Peak inverse voltage, PIV	V_m	$2V_m$	V_m
8.	Crest Factor, CF	$C.F = \frac{I_{sp}}{I_s} = \frac{I_m \times 2}{I_m} = 2$	$\sqrt{2}$	$\sqrt{2}$
9.	Number of diodes	1	2	4
10.	Ripple frequency	f	$2f$	$2f$
11.	Form Factor	$FF = \frac{V_{or}}{V_0} = 1.5708$	1.11	1.11
12.	Peak value of source current	$I_{sp} = I_m$	$I_{sp} = I_m$	$I_{sp} = I_m$
13.	Rms value of source current	$I_s = I_m/2$	$I_s = I_m/\sqrt{2}$	$I_s = I_m/\sqrt{2}$

THREE PHASE RECTIFIERS

The highest possible value of average output voltage from a 1ϕ full wave rectifier is $\frac{2V_m}{\pi} = 0.63662 V_m$. Single phase rectifiers are suitable up to power loads of about 15kw. For higher power demands, 3ϕ rectifiers are preferred due to the following reasons:

(i) Higher dc voltage

(ii) Better TUF

(iii) Better input power factor

(iv) Lesser ripple content in output current. Therefore better load performance.

v) Lower size of filter circuit parameters.

3ϕ rectifier are classified as under:

(a) 3ϕ Half wave rectifier

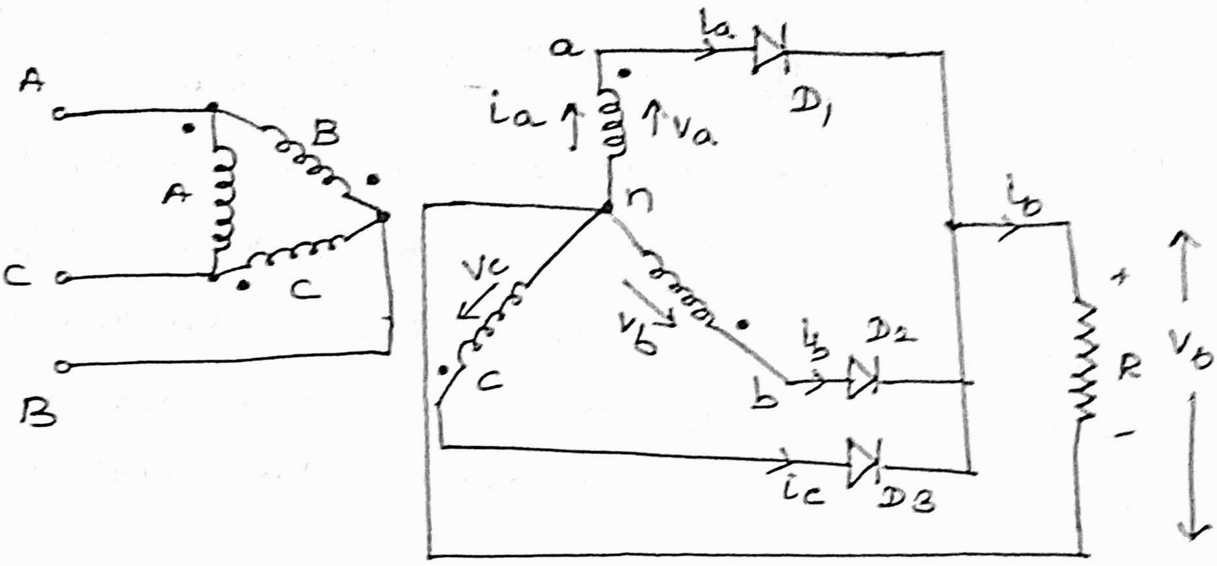
(b) 3ϕ mid point 6 pulse rectifier.

(c) 3ϕ Bridge rectifier.

(d) 3ϕ 12 pulse rectifier.

(a) 3ϕ Half wave rectifier:

It uses 3ϕ Transformer with primary in delta and secondary in star. The 3 diodes D1, D2, D3 one in each phase, have their cathode connected together to common load R. Neutral is used to complete the path for the return of load current.

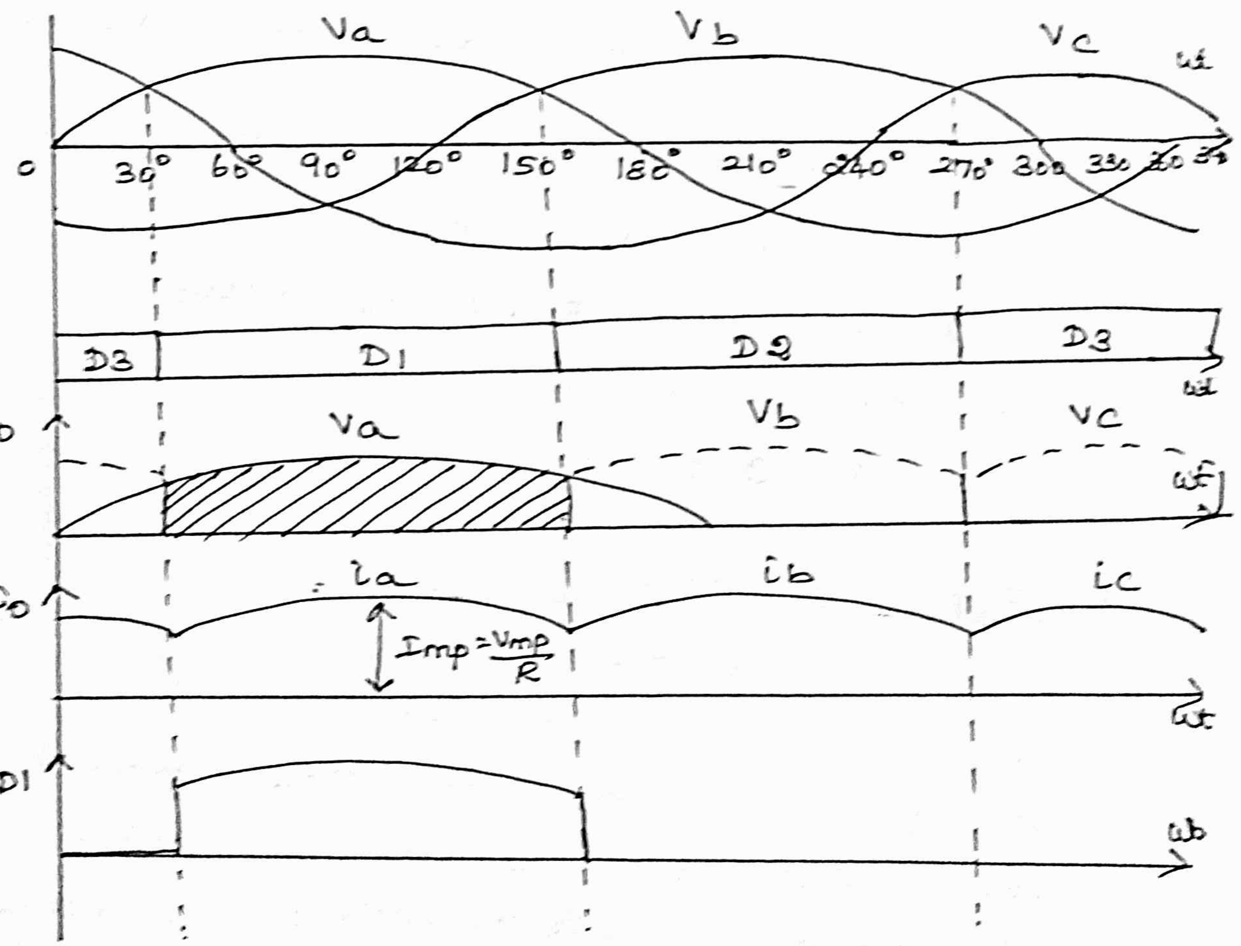


3φ Half wave diode rectifier with common cathode Arrangement.

Diode D_1 conduct for $\omega t = 30^\circ$ to $\omega t = 150^\circ$ Sense the most positive voltage.

Diode D_2 conducts for $\omega t = 150^\circ$ to 270°.

Diode D3 conducts for $\omega t = 270^\circ$ to 390°.



average value $V_0 = 3 \times \frac{1}{2\pi} \int_{\pi/6}^{5\pi/6} V_{mp} \sin \omega t \cdot d(\omega t)$.

$$= \frac{V_{mp} \times 3}{2\pi} \left[-\cos \omega t \right]_{\pi/6}^{5\pi/6}$$

$$= \frac{V_{mp} \times 3}{2\pi} \left[-\cos \frac{5\pi}{6} + \cos \pi/6 \right]$$

$$= \frac{V_{mp} \times 3}{2\pi} \left[+0.8660 + 0.8660 \right]$$

$$= \frac{V_{mp} \times 3 \times 1.732}{2\pi} = \boxed{\frac{3\sqrt{3}}{2\pi} V_{mp}}$$

$V_{mp} \rightarrow$ maximum value of phase voltage,

$V_{mp} = \sqrt{2}\, V_{ph}$.

$$= \frac{3\sqrt{3}}{2\pi} \times \sqrt{2}\, V_{ph} = \boxed{\frac{3\sqrt{6}}{2\pi} V_{ph}}$$

V_{ml} = maximum value of line voltage,

$V_{ml} = \sqrt{3} \times V_{mp} = \sqrt{6}\, V_{ph}$.

$$= \boxed{\frac{3}{2\pi} V_{ml}}$$

$$\frac{3\sqrt{3}}{2\pi} V_{mp} = \frac{3\sqrt{6}}{2\pi} V_{ph} = \frac{3}{2\pi} V_{ml}.$$

Rms value of output voltage

$$V_{or} = \left[\frac{3}{2\pi} \int_{\pi/6}^{5\pi/6} (V_{mp} \sin \omega t)^2 \cdot d\omega t \right]^{1/2}.$$

$= 3/\sqrt{\,}$ ~~Vmp~~ $0.84068\, V_{mp}$.

Ripple voltage $V_r = \sqrt{V_{or}^2 - V_o^2}$

$= V_{mp}\sqrt{0.84068^2 - 0.827^2}$

$= 0.151\ V_{mp}$.

Voltage Ripple factor VRF $= \dfrac{V_r}{V_o} = \dfrac{0.151 V_{mp}}{0.827 V_{mp}}$

$= 0.1826$ (or) 18.26%

Fill Factor $= \dfrac{V_{or}}{V_o} = \dfrac{0.84068}{0.827}$

$= 1.0165$.

Rms value of output current $I_{or} = \dfrac{V_{or}}{R}$

$= \dfrac{0.84068}{R} V_{mp}$

$= 0.84068\ I_{mp}$.

$I_{mp} = \dfrac{V_{mp}}{R}$ → Peak value of load or output current.

$P_{dc} = V_o I_o = \dfrac{3\sqrt{3}}{2\pi} V_{mp} \times \dfrac{3\sqrt{3}}{2\pi} I_{mp}$.

$P_{ac} = V_{or} \cdot I_{or} = (0.84068)^2 V_{mp} I_{mp}$.

Rectifier Efficiency $= \dfrac{P_{dc}^2}{P_{ac}^2} = \left(\dfrac{3\sqrt{3}}{2\pi}\right)^2 \times \dfrac{1}{0.84068^2}$

$= 0.96765$.

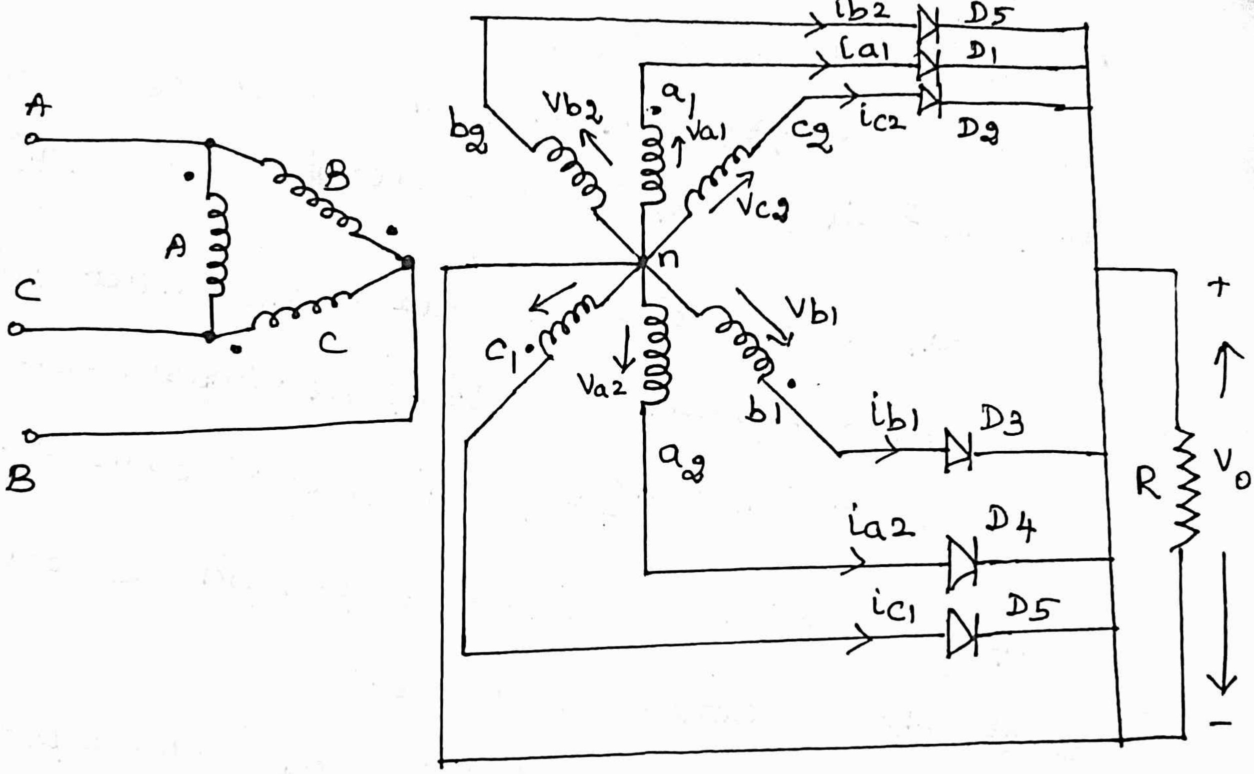
Rms value of Source current $I_s = \left[\dfrac{1}{2\pi}\int_{\pi/6}^{5\pi/6}(I_{mp}\sin\omega t)^2\, d\omega t\right]^{1/2}$

$= 0.4854\ I_{mp}$.

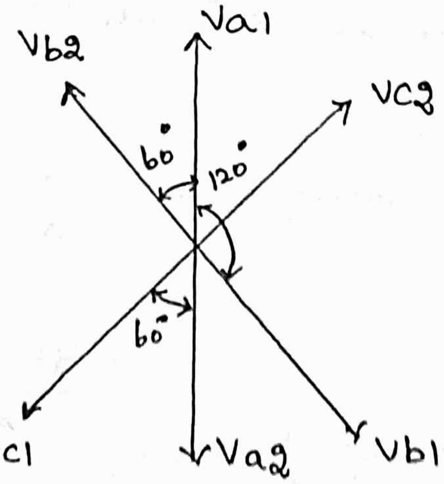
Rms value of source voltage $V_s = \dfrac{V_{mp}}{\sqrt{2}} = 0.707 V_{mp}$

Three Phase Bridge Rectifier

3ϕ Midpoint 6 pulse Diode Rectifier



3 ϕ mid point 6 pulse diode Rectifier



Six phase Voltages

$\Rightarrow$ This rectifier is also called 6 phase half wave diode rectifier (or) 3ϕ M-6 diode rectifier.
diode rectifier (or) 3ϕ M-6 diode rectifier using 6 diodes. 3ϕ

$\Rightarrow$ M-6 diode rectifier using 6 diodes. 3ϕ Transformer with primary in delta and secondary in double star is used.

⇒ 6 phase supply is available from 6 terminals $a_1, c_2, b_1, a_2, c, \& b_2$.

⇒ Phase voltages V_{a1}, V_{b1}, V_{c1} are phase displaced by 120°, $|||^{ly}$ V_{a2}, V_{b2}, V_{c2} are displaced by 120°.

$$V_{a1} = V_{mp} \sin \omega t \quad ; \quad V_{a2} = V_{mp} \sin(\omega t - 180^\circ) = -V_{a1}.$$

$$V_{b1} = V_{mp} \sin(\omega t - 120^\circ) ; \quad V_{b2} = V_{mp} \sin(\omega t - 300^\circ) = -V_{b1}.$$

$$V_{c1} = V_{mp} \sin(\omega t - 240^\circ) ; \quad V_{c2} = V_{mp} \sin(\omega t - 60^\circ).$$

V_{mp} → Maximum value of per phase voltage.

⇒ Each diode conducts for 60°.

⇒ From $\omega t = 0^\circ$ to $\omega t = 60^\circ$, V_{b2} highest positive, So D_6 conducts.

From $\omega t = 60^\circ$ to $\omega t = 120^\circ$, V_{a1} highest positive, So D_1 conducts.

Average output voltage $V_0 = \frac{3}{\pi} \int_{\pi/3}^{2\pi/3} V_{mp} \sin \omega t \cdot d(\omega t)$.

$$= \frac{3 V_{mp}}{\pi}$$

Rms value of output voltage,

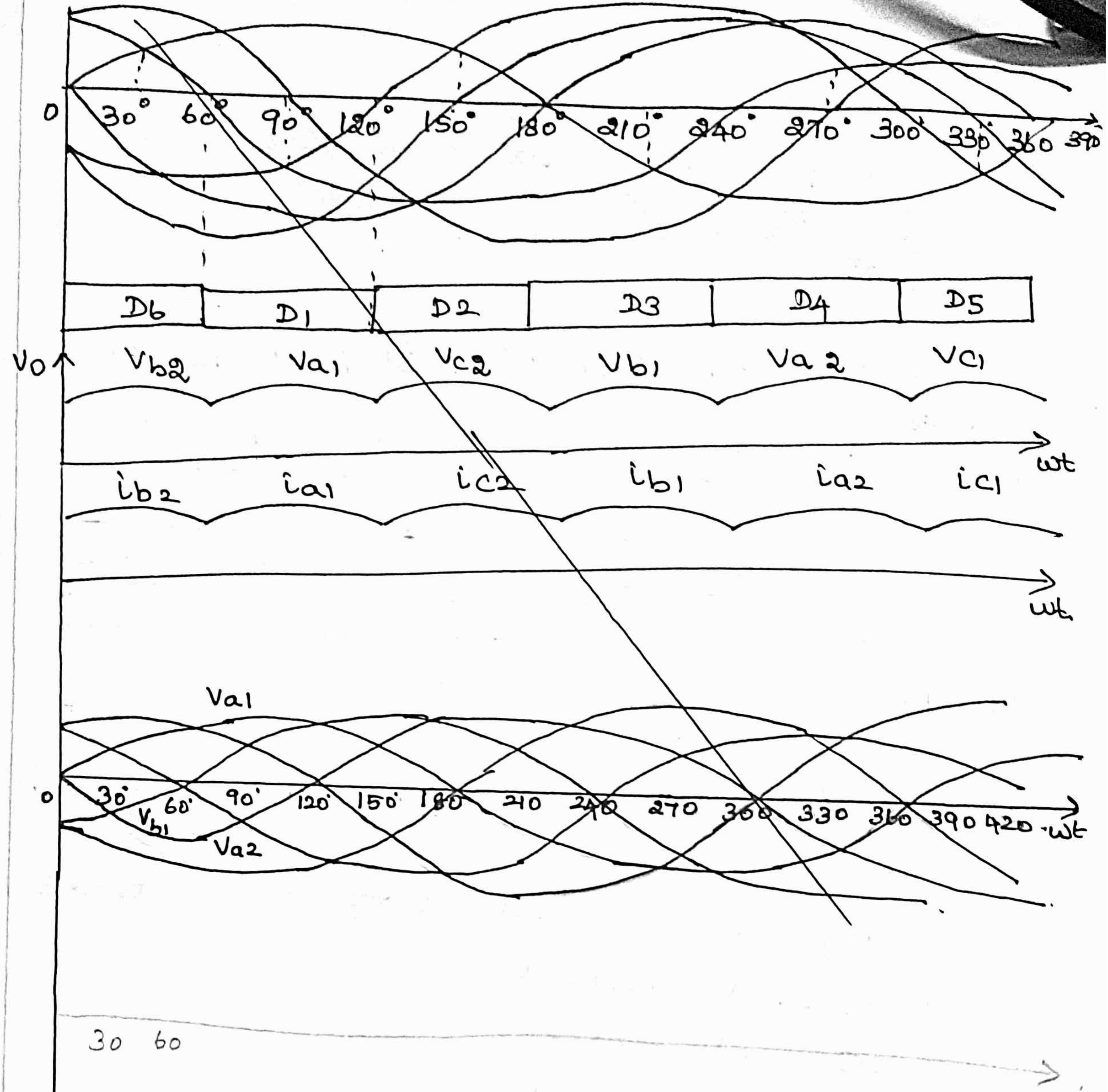
$$V_{or} = \left[\frac{3}{\pi} \int_{\pi/3}^{2\pi/3} (V_{mp} \sin^2 \omega t) d\omega t \right]^{1/2}$$

$$= 0.9558 V_{mp}.$$

Ripple voltage $V_r = \sqrt{V_{or}^2 - V_0^2} = 0.0408 V_{mp}$.

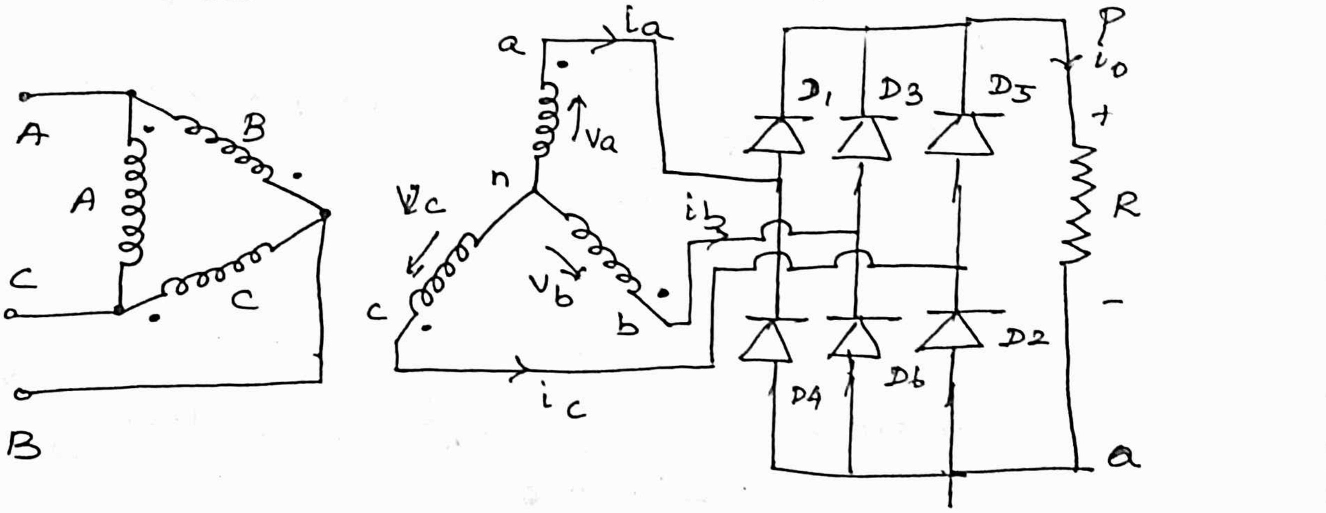
$$VRF = \frac{V_r}{V_0} = 0.043.$$

$$FF = \frac{V_{or}}{V_0} = \frac{0.9558 \times \pi}{3} = 1.009.$$


0
30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 360 390
D6 D1 D2 D3 D4 D5
Vo
Vb2 Va1 Vc2 Vb1 Va2 Vc1
ωt
ib2 ia1 ic2 ib1 ia2 ic1
ωt
Va1
0
30° 60° 90° 120° 150° 180° 210 240 270 300 330 360 390 420 ωt
Vb1
Va2
30 60

Three Phase Bridge Rectifier

3 phase bridge rectifier using 6 diodes. The diodes are arranged in 3 legs. Each leg has two series connected diodes. Upper diodes D_1, D_3, D_5 constitute the positive group of diodes. The lower diodes D_2, D_4, D_6 form the negative group of diodes. This rectifier is also called 3 phase 6 pulse diode rectifier (or) 3 phase full wave diode rectifier (or) 3ϕ B-6 diode rectifier.



3ϕ Bridge Rectifier using Diodes.

⇒ From $\omega t = 30°$ to $150°$, V_a is more positive than the voltages V_b, V_c. Therefore D_1 conducts.

⇒ From $\omega t = 150°$ to $270°$, V_b is more positive, compared to V_a, V_c. Therefore diode D_3 conducts.

⇒ From $\omega t = 270°$ to $390°$, D_5 conducts.

⇒ From $\omega t = 90°$ to $210°$, D_2 conducts. (most negative)

⇒ From $\omega t = 210°$ to $330°$, D_4 conducts.

⇒ From $\omega t = 330°$ to $450°$. D_6 conducts and so on.

Average value of load voltage

$$V_0 = \frac{3}{\pi} \int_{\pi/6}^{\pi/2} V_{ml} \sin(\omega t + 30^\circ)\, d\omega t.$$

$$= \frac{3 V_{ml}}{\pi} = \frac{3\sqrt{2} V_L}{\pi} = \frac{3\sqrt{6} V_P}{\pi}$$

$V_{ml} \Rightarrow$ maximum value of line voltage.

$V_L \rightarrow$ rms value of line voltage.

$V_P \rightarrow$ rms value of phase voltage.

$$V_0 = \frac{3}{\pi} \int_{\pi/6}^{\pi/2} V_{ml} \sin(\omega t + 30^\circ)\, d\omega t.$$

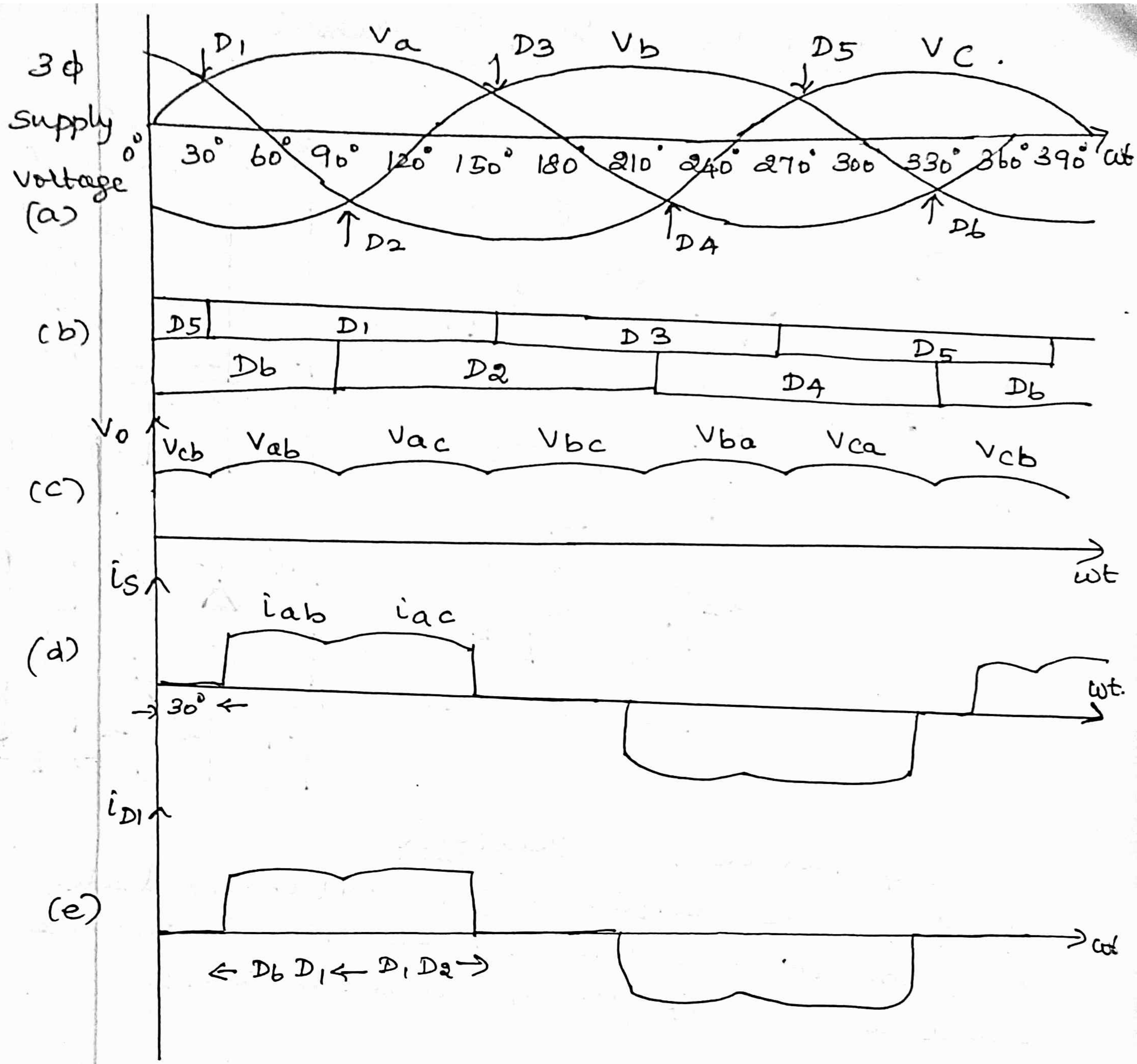
$$= \frac{3 V_{ml}}{\pi}.$$

$$V_{0\,rms} = \left[\frac{3}{\pi} \int_{\pi/3}^{2\pi/3} V_{ml}^2 \sin^2 \omega t \cdot d(\omega t) \right]^{1/2}.$$

$$= 0.9558 V_{ml}.$$

Ripple voltage $V_r = \sqrt{V_{or}^2 - V_0^2}$

$$= 0.0408 V_{ml}.$$



(a) → 3 ϕ input voltage waveform.

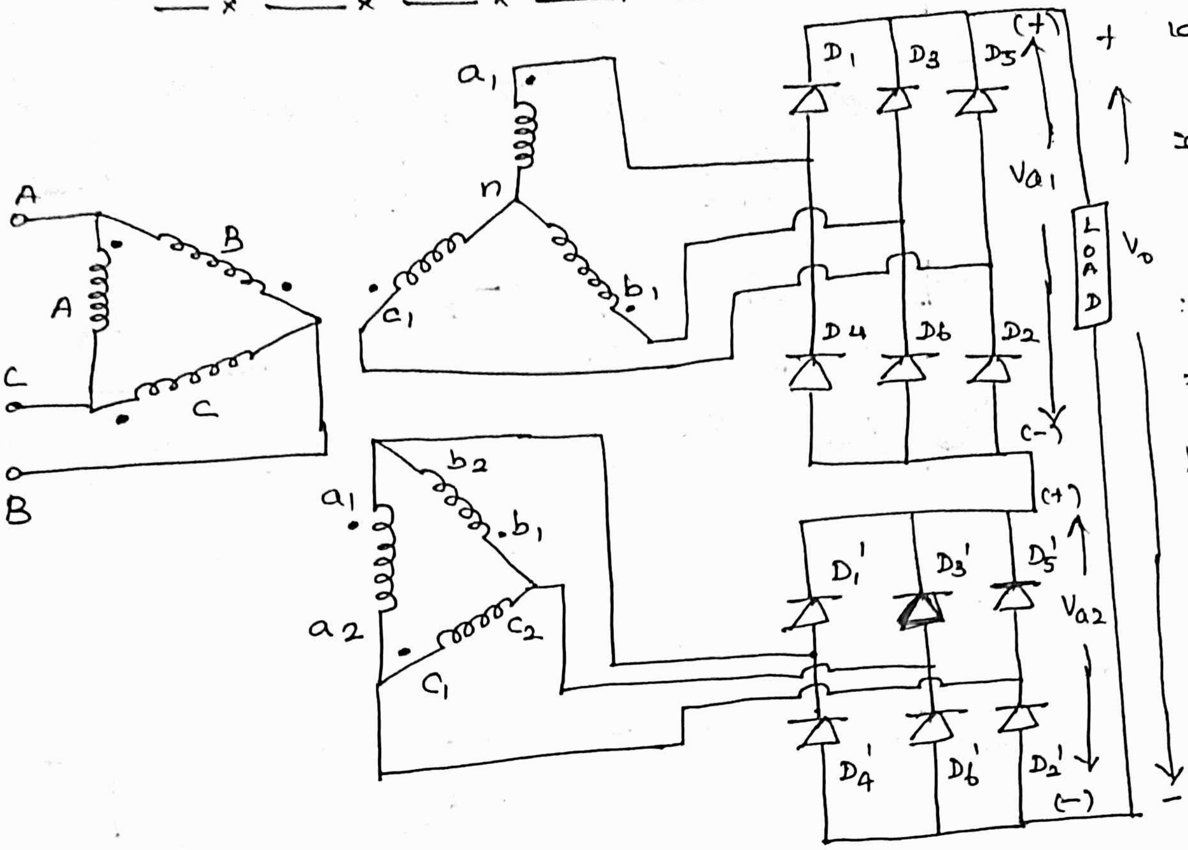
(b) → conduction sequence of diodes.

(c) → output voltage waveform.

(d) → input current waveform.

(e) → Diode current waveform through D_1.

Three Phase Twelve pulse Rectifier



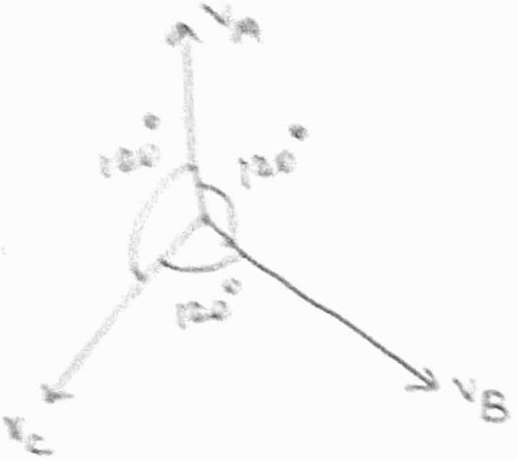
3 ϕ Twelve Pulse Rectifier

$\Rightarrow$ 3 ϕ 12 pulse rectifier using a total of 12 diodes.

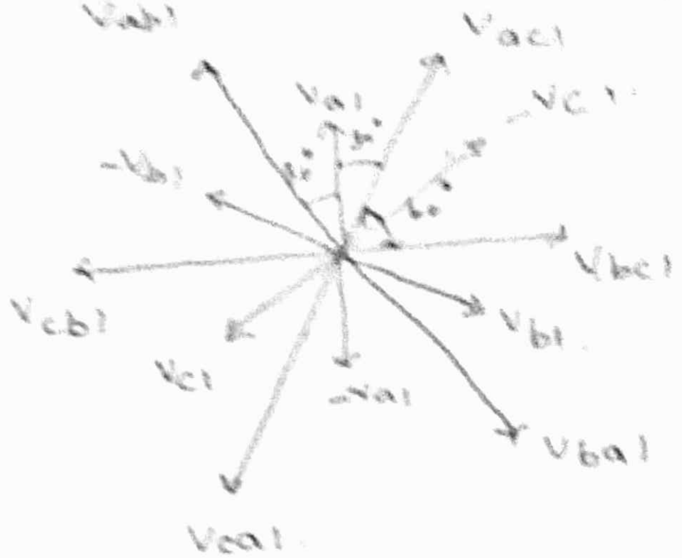
A 3ϕ transformer with two secondaries and one Delta connected primary feeds the rectifier circuit.

* one secondary winding is connected in star and the other is in delta.
* star connected secondary feeds the upper 3 phase diode bridge rectifier 1, whereas the delta connected secondary is connected to lower 3 phase diode bridge rectifier 2.

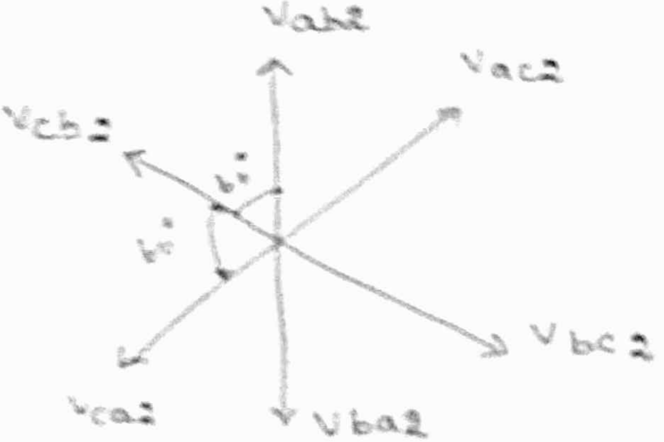
net output or load voltage = output voltage of upper rectifier V_{01} + output voltage of lower rectifier V_{02}.



Primary line voltage



Line voltages for secondary star



Line voltage for secondary Delta.

Average value of output voltage

$$V_o = \frac{6}{\pi}\int_{75^\circ}^{105^\circ} V_p \sin\omega t \, d(\omega t) = 0.988616 V_p.$$

$$= 0.988616 \times 1.932 V_{ml} = 1.91 V_{ml}$$

Rms value of output voltage,

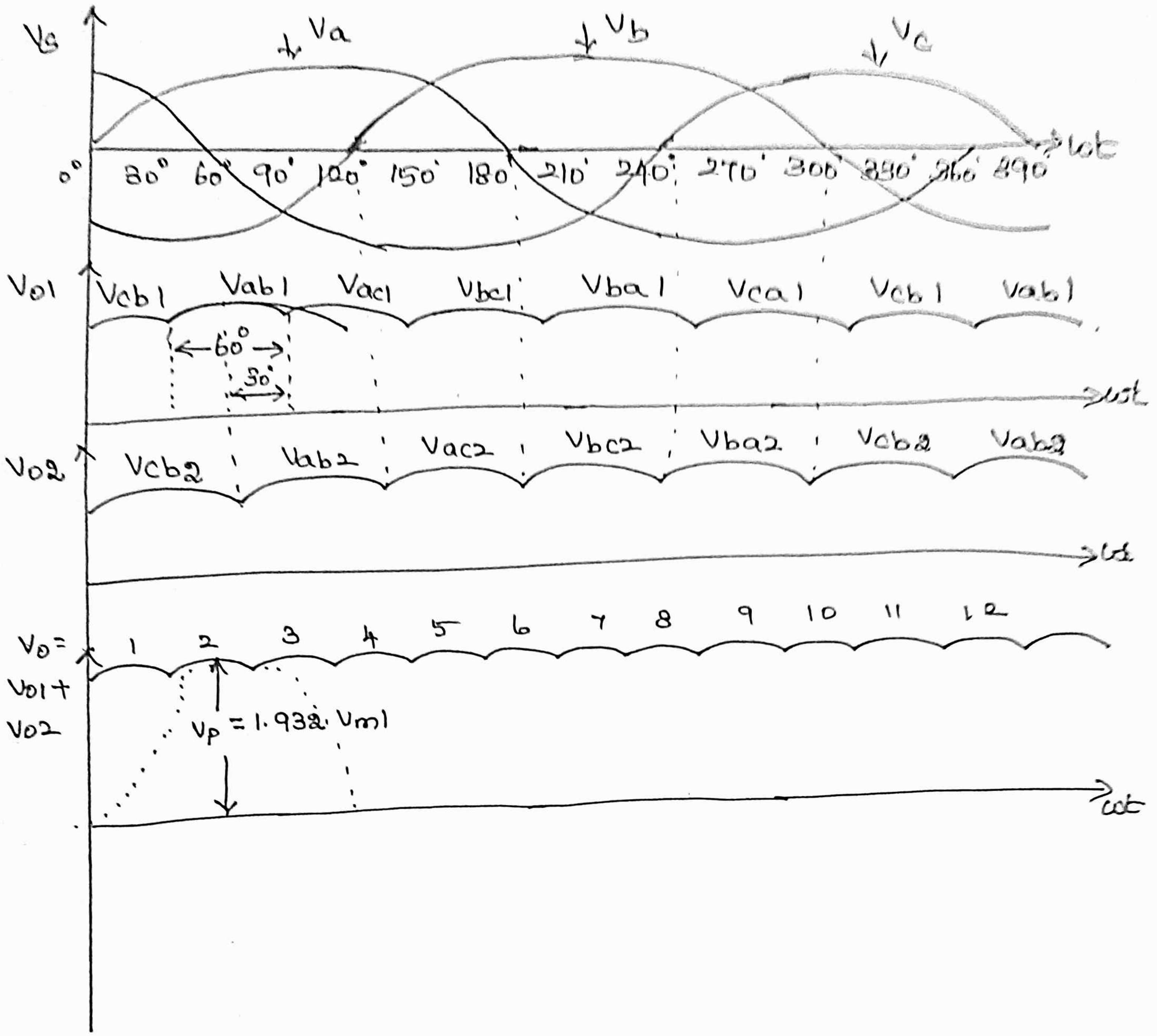
$$V_{or} = \left[\frac{6}{\pi}\int_{75^\circ}^{105^\circ} V_p^2 \sin^2 \omega t \, . \, d(\omega t)\right]^{1/2}$$

$$= 0.988668 V_p.$$

$$= 0.988668 \times 1.932 V_{ml} = 1.9101 V_{ml}.$$

Ripple voltage $V_r = \sqrt{V_{or}^2 - V_o^2} = \left[(1.910)^2 - (1.91)^2\right]^{1/2} V_{ml}$

$$\boxed{V_r = 0.01954 5 V_{ml}}.$$



Vs
Va
Vb
Vc
ωt
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°
390°
Vo1
Vcb1
Vab1
Vac1
Vbc1
Vba1
Vca1
Vcb1
Vab1
60°
30°
ωt
Vo2
Vcb2
Vab2
Vac2
Vbc2
Vba2
Vcb2
Vab2
ωt
Vo =
Vo1 +
Vo2
1 2 3 4 5 6 7 8 9 10 11 12
Vp = 1.932 Vm1
ωt

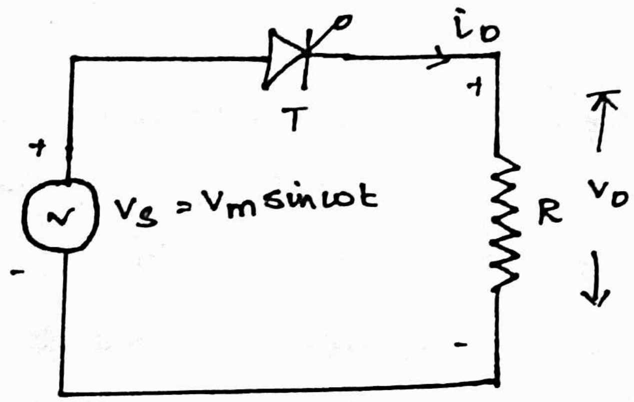
UNIT - II

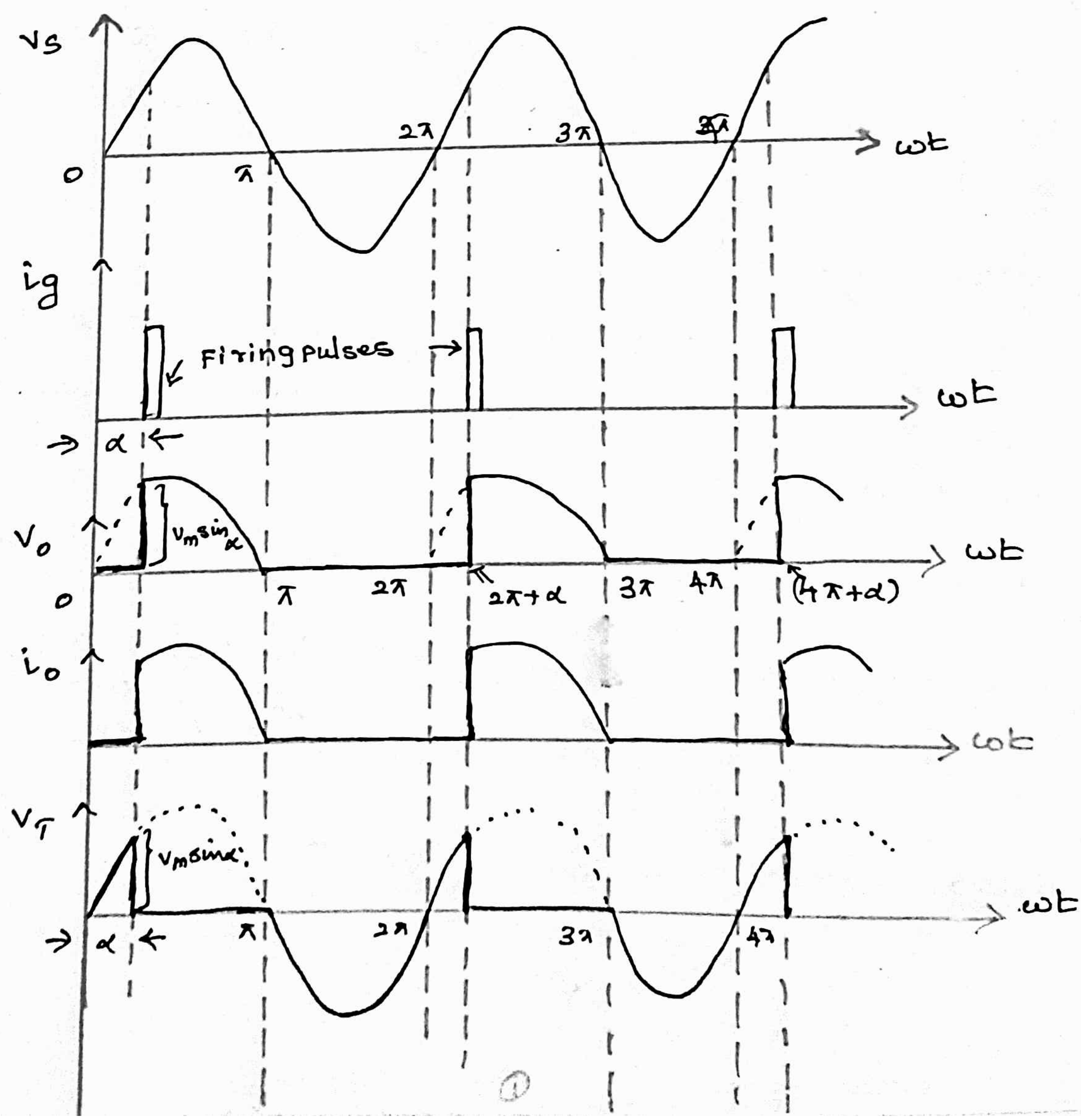
PHASE - CONTROLLED CONVERTERS

2-pulse, 3-pulse and 6-pulse converters - performance Parameters - Effect of source inductance - Gate circuit schemes for phase control - Dual converters.

Single phase half-wave thyristor circuit with R Load :

2, 23, 26,





Circuit operation :-

An SCR can conduct only when anode voltage is positive and a gating signal is applied. At some delay angle α, a positive gate signal applied between gate and cathode turns ON the SCR. Full supply voltage is applied to the load as V_o.

Firing angle :-

A firing angle may be defined as the angle measured from the instant SCRs gets forward biased to the instant it is triggered.

Once the SCR is on, load current flows, it is turned-off by reversal of voltage. At $\omega t = \pi, 3\pi, 5\pi$ etc. load current falls to zero, supply voltage reverse biases the SCR, the device is turned off.

phase control :-

phase relationship between the start of the load current and the supply voltage can be controlled.

Thyristor remains ON from $\omega t = \alpha$ to π, $(2\pi+\alpha)$ to 3π etc.

It is off from π to $(2\pi+\alpha)$, 3π to $(4\pi+\alpha)$ etc.

circuit turn off time $t_c = \frac{\pi}{\omega}$ sec.

Average voltage V_o across load R

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \cdot \sin\omega t \cdot d(\omega t).$$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi}\left[-\cos\pi - (-\cos\alpha)\right].$$

$\because \cos\pi = -1.$

$$= \frac{V_m}{2\pi}\left[-(-1) + \cos\alpha\right]$$

$$V_o = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]$$

$$V_{o\,r.m.s} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t \; (d\omega t)\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi}\int_{\alpha}^{\pi} \sin^2\omega t \; d(\omega t)\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi}\int_{\alpha}^{\pi} \frac{1-\cos 2\omega t}{2} \,.\, d(\omega t)\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi \cdot 2}\int_{\alpha}^{\pi} 1-\cos 2\omega t \,.\, d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}\right]^{1/2}$$

$\because \int \cos 2\omega t = \frac{\sin 2\omega t}{2}.$

$$= \frac{V_m}{2\sqrt{\pi}}\left[\pi - \frac{\sin 2\pi}{2} - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$V_{o\,r.m.s} = \frac{V_m}{2\sqrt{\pi}}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

The value of r.m.s current $I_{o.r}$ is

$$I_{o.r.m.s} = \frac{V_{o\,r.m.s}}{R} = \frac{V_m}{2R\sqrt{\pi}}\left[\pi - \alpha + \frac{1}{2}\sin 2\alpha\right]^{1/2}.$$

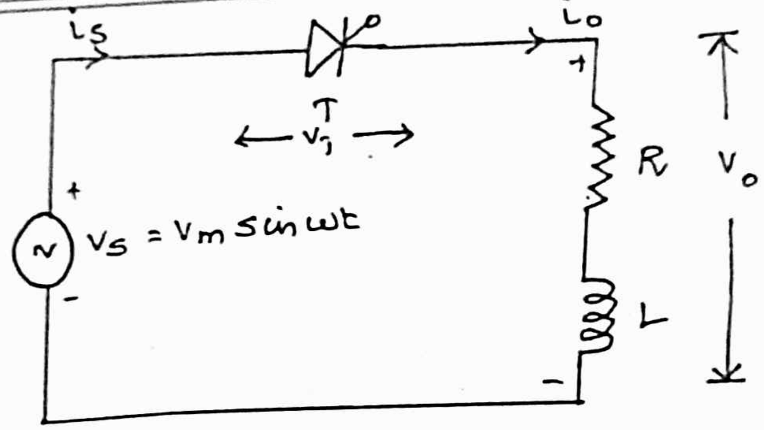
Input volt amperes = rms source voltage × total rms line current

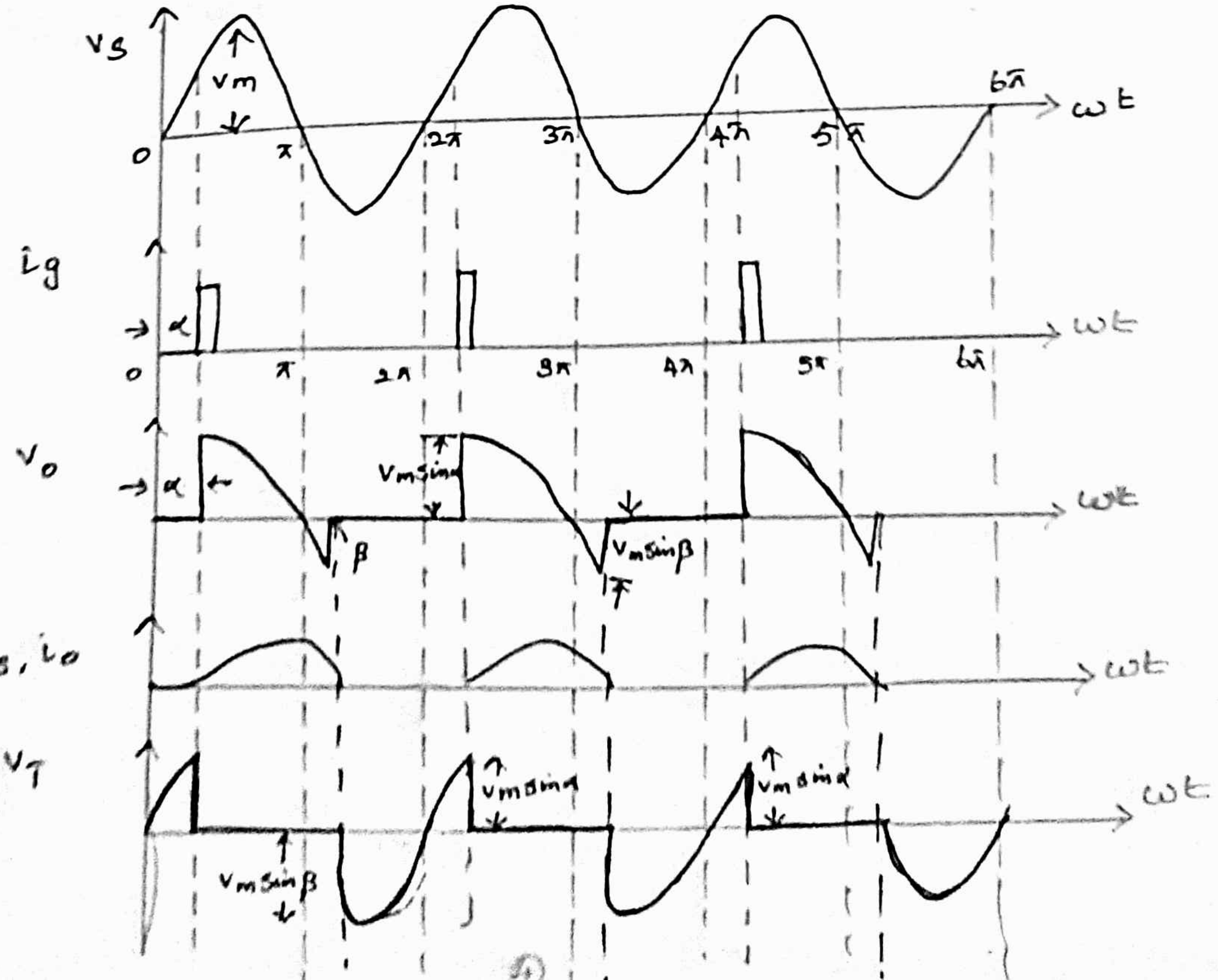
$= V_s \cdot I_{or}$

Input power factor $= \dfrac{\text{Power delivered to load}}{\text{Input VA}}$

$= \dfrac{V_{or}}{V_s}$.

Single phase Half wave circuit with RL load :-





3.

circuit operation :-

At $\omega t = \alpha$, thyristor is turned ON by gating signal. Inductance L forces the load or output current i_o to rise gradually. After some time, i_o reaches maximum value and then begins to decrease. At $\omega t = \pi$, V_o is zero, but i_o is not zero because of the load inductance.

After $\omega t = \pi$, SCR is subjected to reverse anode voltage but it will not be turned off as load current i_o is not less than the holding current. At some angle $\beta > \pi$, i_o reduces to zero, SCR is turned off as it is already reverse biased. After $\omega t = \beta$, $V_o = 0$, $i_o = 0$.

Angle β is called the extinction angle

$\beta - \alpha = \gamma$ is called the conduction angle.

circuit turn-off time $t_c = \dfrac{2\pi - \beta}{\omega}$.

Average load voltage $V_o = \dfrac{1}{2\pi} \displaystyle\int_{\alpha}^{\beta} V_m \cdot \sin\omega t \, d(\omega t)$.

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\beta}$$

$$= \frac{V_m}{2\pi}\left[-\cos\beta + \cos\alpha\right]$$

$$V_o = \frac{V_m}{2\pi}\left[\cos\alpha - \cos\beta\right].$$

$$V_{o\,r.m.s} = \left[\frac{1}{2\pi}\int_{\alpha}^{\beta} V_m^2 \cdot \sin^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\pi}}\left[\frac{1}{2}\int_{\alpha}^{\beta} \frac{1-\cos 2\omega t}{2} \, d(\omega t)\right]^{1/2}$$

$$\therefore \sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left\{\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\beta}\right\}^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[\beta - \frac{\sin 2\beta}{2} - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[(\beta-\alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{1/2}.$$

The load current i_o consists of two components.

i) steady state component (i_s)

ii) Transient component (i_t)

where $i_s = \dfrac{V_m}{\sqrt{R^2+x^2}} \sin(\omega t - \phi)$.

$$\phi = \tan^{-1}\frac{x}{R} \quad ; \quad x = \omega L$$

$$i_t = A \cdot e^{-(R/L)t}$$

$$i_o = i_s + i_t = \frac{V_m}{2}\sin(\omega t - \phi) + A e^{-(R/L)t} \quad \rightarrow (1).$$

Constant A can be obtained from boundary condition at $\omega t = \alpha$.

$$t = \frac{\alpha}{\omega}, \quad i_o = 0.$$

sub these values in eqn (1)

$$0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A e$$

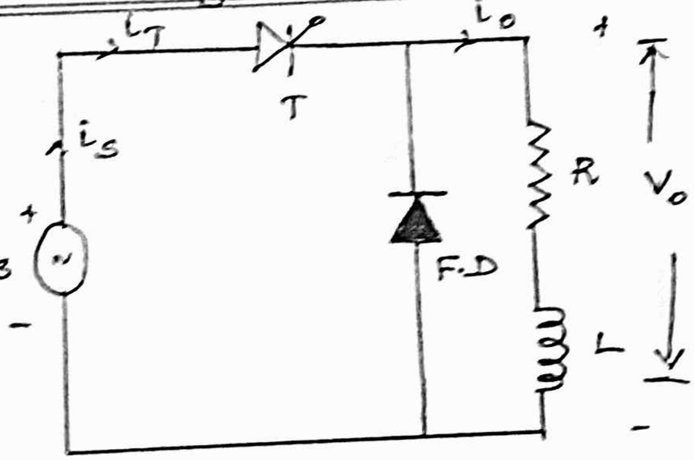
$$A = -\frac{V_m}{Z} \sin(\alpha - \phi) \cdot e^{\frac{R\alpha}{\omega L}}.$$

Sub the value of A in eqn ①

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{V_m}{Z} \sin(\alpha - \phi)$$

$$\exp\left\{-\frac{R}{\omega L}(\omega t - \alpha)\right\}.$$

Single phase half wave circuit with RL load and freewheeling diode:



circuit operation :-

At $\omega t = 0$, source voltage is becoming positive. At some delay angle α, forward biased SCR is triggered and source voltage V_s appears across load as V_o.

At $\omega t = \pi$, source voltage V_s is zero, freewheeling diode FD is forward biased. Load current i_o is transferred from SCR to FD. It is assumed that during freewheeling period load current does not decay to zero, until the SCR is triggered again at $(2\pi + \alpha)$.

There are two modes of operation

i) <u>conduction mode</u> : $\alpha \leq \omega t \leq \pi$

SCR conducts from α to π, $2\pi + \alpha$ to 3π and so on.

The voltage equation

$$V_m \sin \omega t = R i_0 + L \cdot \frac{di_0}{dt}.$$

$$i_0 = \frac{V_m}{Z} \sin(\omega t - \phi) + A e^{-(R/L)t}$$

at $\omega t = \alpha$, $i_0 = I_0$,

$t = \alpha/\omega$, $i_0 = I_0$

$$A = \left[I_0 - \frac{V_m}{Z} \sin(\alpha - \phi)\right] e^{R\alpha/\omega L}$$

$$i_0 = \frac{V_m}{Z} \sin(\omega t - \phi) + \left[I_0 - \frac{V_m}{Z} \sin(\alpha - \phi)\right] \left\{e^{-R/L\left(t - \frac{\alpha}{\omega}\right)}\right\}$$

ii) <u>Free wheeling mode</u> : $\pi < \omega t \leq (2\pi + \alpha)$.

FD conducts from π to $2\pi + \alpha$, 3π to $4\pi + \alpha$ and so on.

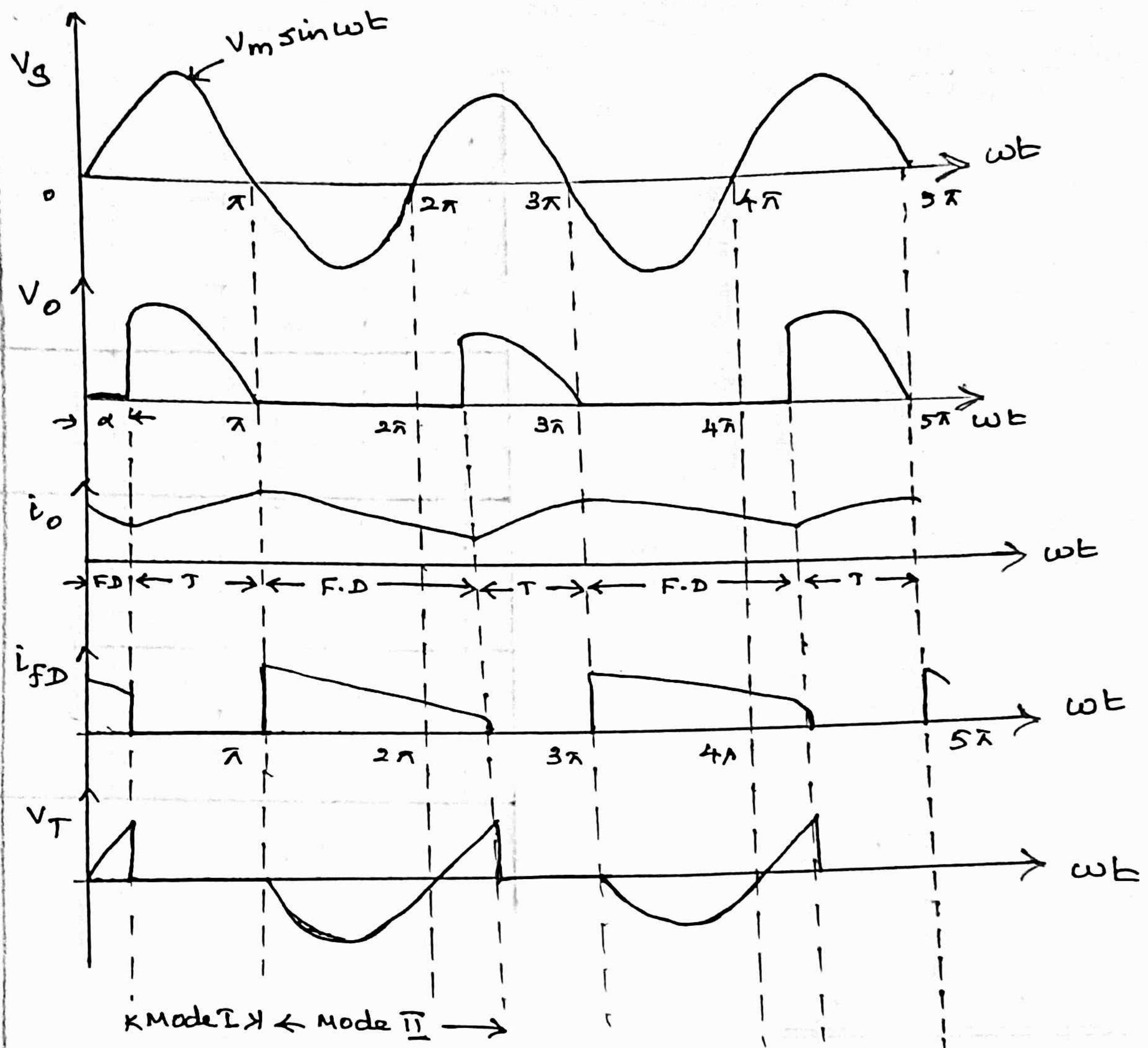
$$i_0 = I_{01} \cdot \exp\left[-R/L\left(t - \pi/\omega\right)\right].$$

average load voltage

$$V_0 = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \, d(\omega t).$$

$$= \frac{V_m}{2\pi} \left[-\cos \omega t\right]_{\alpha}^{\pi}$$

$$V_0 = \frac{V_m}{2\pi} \left[-\cos \pi + \cos \alpha\right] = \frac{V_m}{2\pi} \left[1 + \cos \alpha\right]$$



Load absorbs power for α to π

Energy stored in L is delivered to R for π to $(2\pi+\alpha)$

Advantages of using freewheeling diodes are

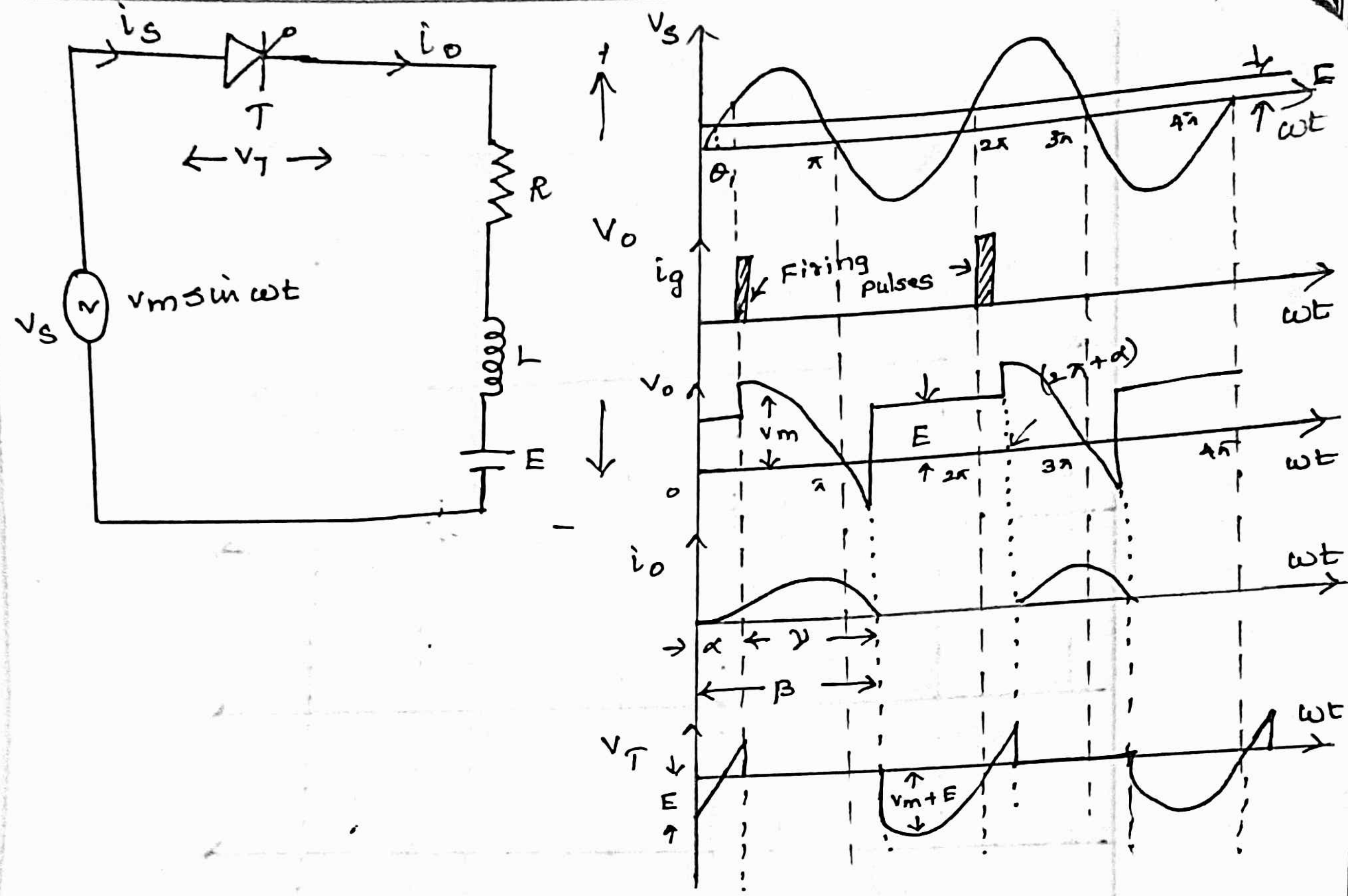
(i) Input power factor is improved.

(ii) Load current waveform is improved.

(iii) load performance is better.

iv) converter efficiency improves.

Single phase Half wave circuit with RLE load



circuit operation :

The counter emf E in the load may be due to a battery or a d.c motor. When SCR T_1 is fired at an angle $\alpha < \theta_1$, then $E > V_s$, SCR is reverse biased. It will not turn ON. With SCR ON, the voltage equation is :

$$V_m \sin \omega t = R i_o + L \cdot \frac{di_o}{dt} + E$$

The solution of the above equation has two components:

i) steady state current component (i_s)

ii) Transient current component (i_t).

$$i_{s1} = \frac{V_m}{Z} \sin(\omega t - \phi)$$

$$i_{s2} = -(E/R)$$

$$i_t = A e^{-(R/L)t}$$

Total current $i_o = i_{s1} + i_{s2} + i_t$

Average load voltage $V_o = \frac{1}{2\pi} \int_{\alpha}^{\beta} (V_m \sin\omega t - E)\, d(\omega t)$.

$$= \frac{1}{2\pi} \left[-\cos\omega t\, V_m - E \cdot \omega t \right]_{\alpha}^{\beta}.$$

$$= \frac{1}{2\pi} \left[-(\cos\beta + \cos\alpha) V_m - E(\beta - \alpha) \right]$$

Conduction angle $\gamma = \beta - \alpha$,

$$\beta = \alpha + \gamma$$

$$V_o = \frac{1}{2\pi} \left[\cos\alpha - \cos(\alpha + \gamma) - E\gamma \right]$$

using the trigonometric relation,

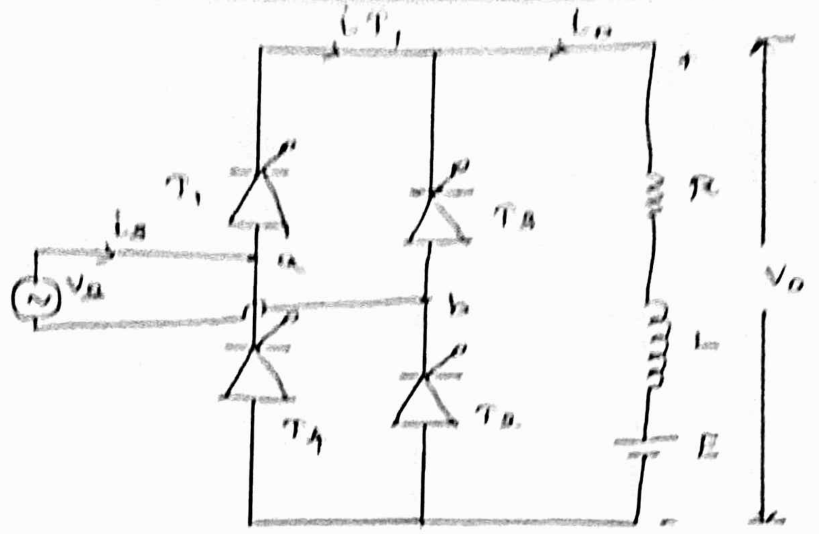
$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}.$$

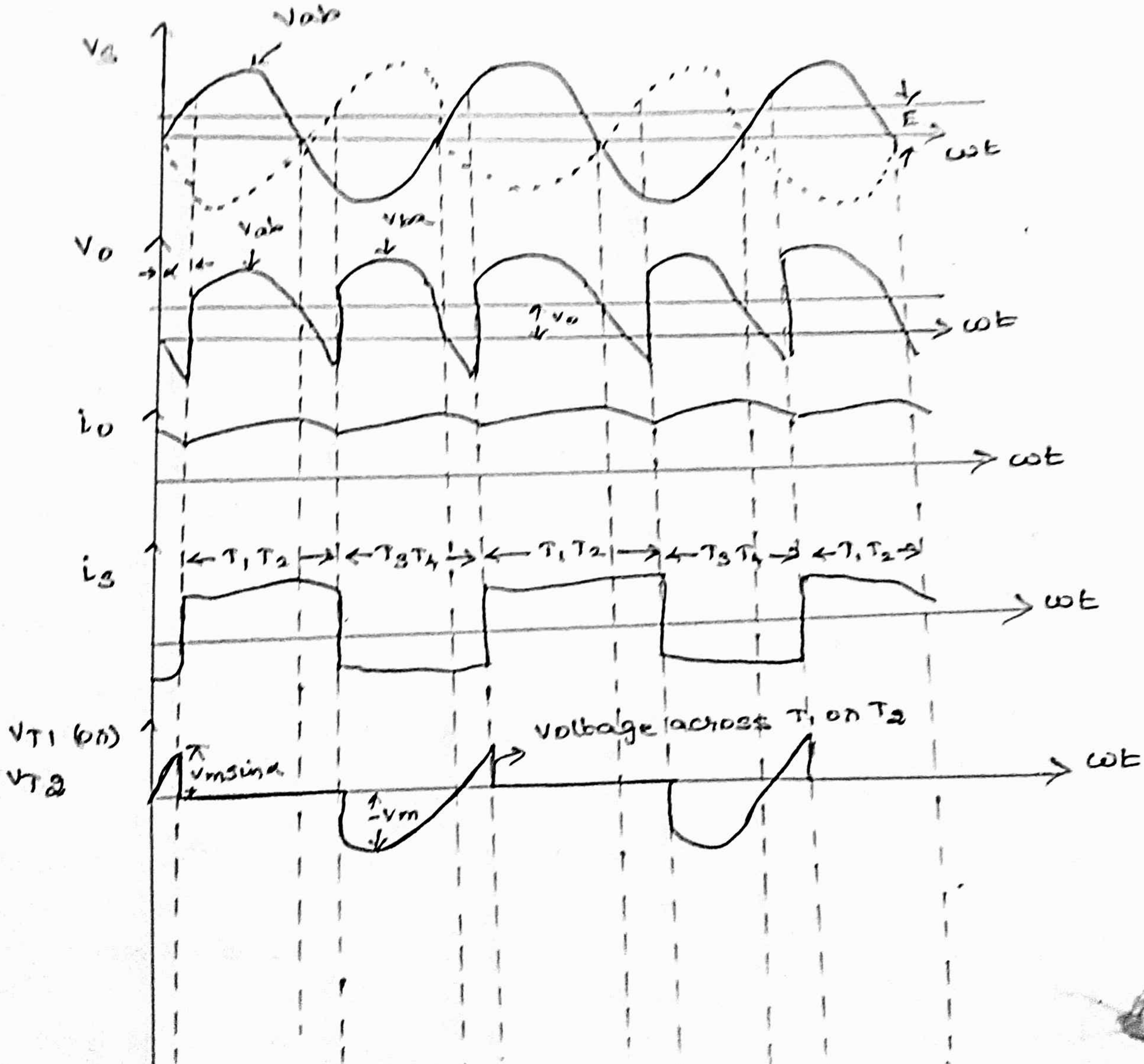
$$V_o = \frac{1}{2\pi} \left[2V_m \sin\left[\frac{2\alpha + \gamma}{2}\right] \sin\frac{\gamma}{2} - E \cdot \gamma \right].$$

$$V_o = \frac{1}{2\pi} \left[2V_m \sin\left(\alpha + \frac{\gamma}{2}\right) \sin\frac{\gamma}{2} - E \cdot \gamma \right].$$

Circuit turn off time $= \frac{2\pi + \theta_1 - \beta_1}{\omega}$ sec.

Single phase full converter bridge with RLE load
[2 pulse converter]





circuit operation :-

A single phase full converter bridge is using four SCRs. The load is assumed to be of RLE type. Thyristor pair T_1, T_2 is simultaneously triggered. when a is positive with respect to B, supply voltage waveform is V_{ab}. when b is positive, with respect to a, supply voltage waveform is V_{ba}.

$V_{ab} = -V_{ba}$.

Forward biased SCRs T_1, T_2 are triggered at $\omega t = \alpha$, they get turned ON. T_3, T_4 reverse bias they are turned off by natural or line commutation.

T_1, T_2 conduct from $\omega t = \alpha$ to $\pi + \alpha$.

T_3, T_4 conduct from $\omega t = \pi + \alpha$ to $2\pi + \alpha$.

During the interval π to $(\pi + \alpha)$, V_s is negative but is is positive. the load therefore returns some of its energy to the supply system.

$$\text{average output voltage } (V_0) = \frac{2\pi}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \cdot \sin\omega t \cdot d(\omega t).$$

$$V_0 = \frac{2V_m}{2\pi} \left[-\cos\omega t \right]_{\alpha}^{\pi+\alpha}$$

$\because \cos(A+B) = \cos A \cos B + \sin A \sin B$

$\sin\pi = 0$

$\cos\pi = -1$

$$= \frac{V_m}{\pi} \left[-\cos(\pi+\alpha) + \cos\alpha \right].$$

$$= \frac{V_m}{\pi} \left[-\left[\cos\pi\cos\alpha + \sin\pi\sin\alpha\right] + \cos\alpha \right].$$

$$= \frac{V_m}{\pi} \left[-(-1)\cos\alpha + \cos\alpha \right].$$

$$V_0 = \frac{2V_m}{\pi} \cos\alpha.$$

(2)

$$V^2_{o\,r.m.s} = \left[\frac{2\times 1}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t \; d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\pi}}\left[\int_{\alpha}^{\pi+\alpha} \frac{1-\cos 2\omega t}{2} \; d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left\{\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi+\alpha}\right\}^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\pi+\alpha - \frac{\sin 2(\pi+\alpha)}{2} - \alpha + \frac{\sin 2\alpha}{2}\right.$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin 2\pi = 0.$

$\cos 2\pi = 1.$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\pi+\alpha - \frac{\sin(2\pi+2\alpha)}{2} - \alpha + \frac{\sin 2\alpha}{2}\right.$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}}\left[\pi - \left(\frac{\sin 2\pi\cos 2\alpha + \cos 2\pi \sin 2\alpha}{2}\right) + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}}\left[\pi - \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}} \times \sqrt{\pi} = \frac{V_m}{\sqrt{2}}$$

Rectification mode :-

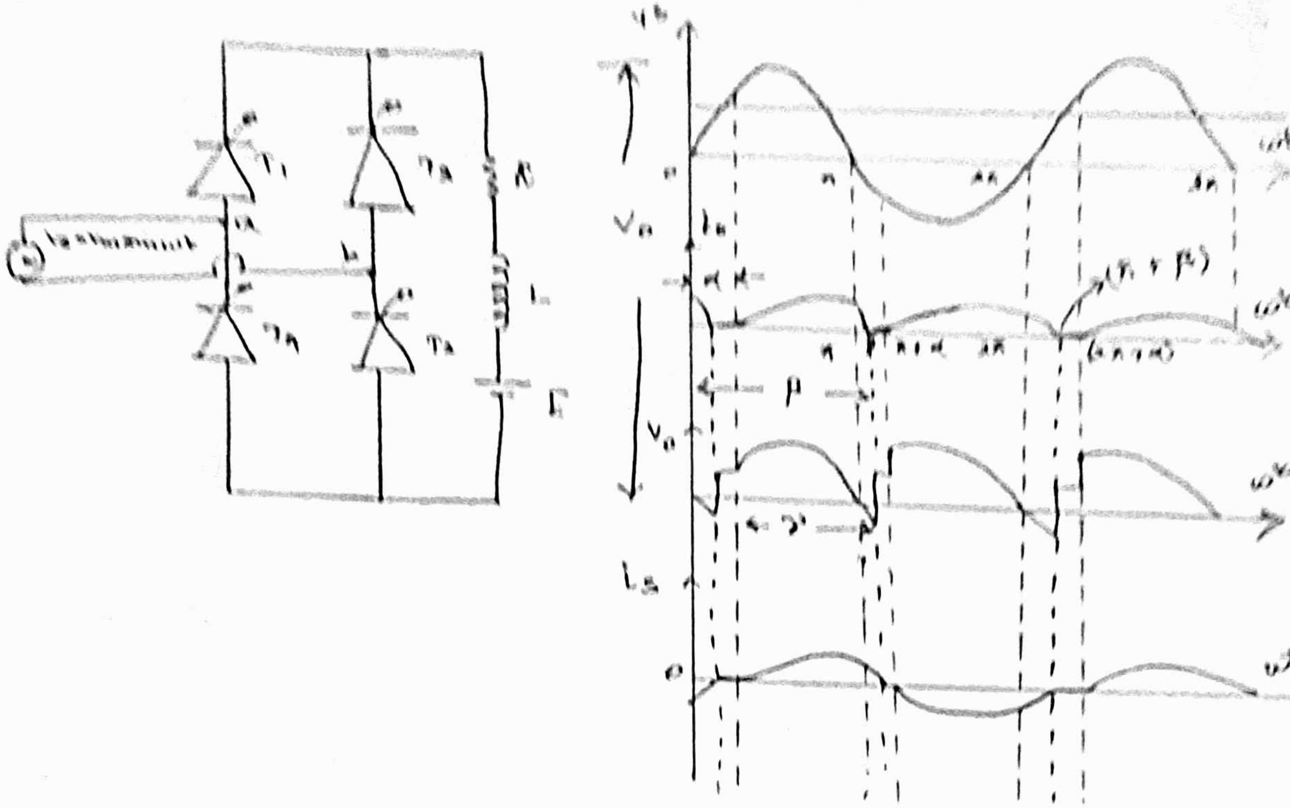
During the period from α to π, input voltage Vs, input current is are positive. Power flows from the supply to the load. The converter is said to be operated in Rectification mode

Inversion mode :-

During the period from π to $\pi+\alpha$, input voltage Vs is negative, input current is is positive

Power flows from the load to the supply. The converter is said to be operated in inversion mode.

Single phase full converter with Discontinuous current.



In practice, the output current may become discontinuous at high values of firing angle or at low values of load current.

Discontinuous means load current reaches zero, during each half cycle before the next SCR in sequence is fired.

Continuous means load current never ceases but continuous to flow through SCR/diode.

circuit operation :

SCR pair $T_1 T_2$ is triggered at $\omega t = \alpha$, load current decays to zero. here $\beta > \pi$.

$T_1 T_2$ are reverse biased after $\omega t = \pi$, this pair is commutated at $\omega t = \beta$ when $i_o = 0$.

<u>conduction period</u>

$\alpha < \omega t < \beta$, T_1, T_2 conduct, $V_o = V_s$.

$(\pi+\alpha) < \omega t < (\pi+\beta)$, T_3, T_4 conduct, $V_o = V_s$.

<u>Idle period</u> :

$\beta < \omega t < (\pi+\alpha)$, no circuit element conducts and $V_o = E$.

Average load voltage $V_o = \frac{1}{\pi} \int_{\alpha}^{\beta} V_m \cdot \sin \omega t \cdot d(\omega t)$

$+ E(\pi + \alpha - \beta)$.

$$= \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\beta} + E(\pi+\alpha-\beta)$$

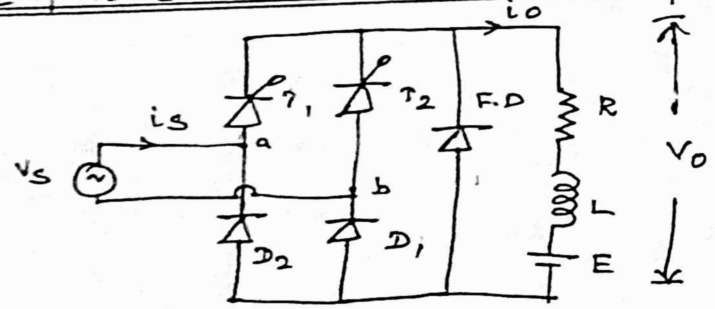
$$= \frac{V_m}{\pi} \left[-\cos\beta + \cos\alpha \right] + E(\pi+\alpha-\beta).$$

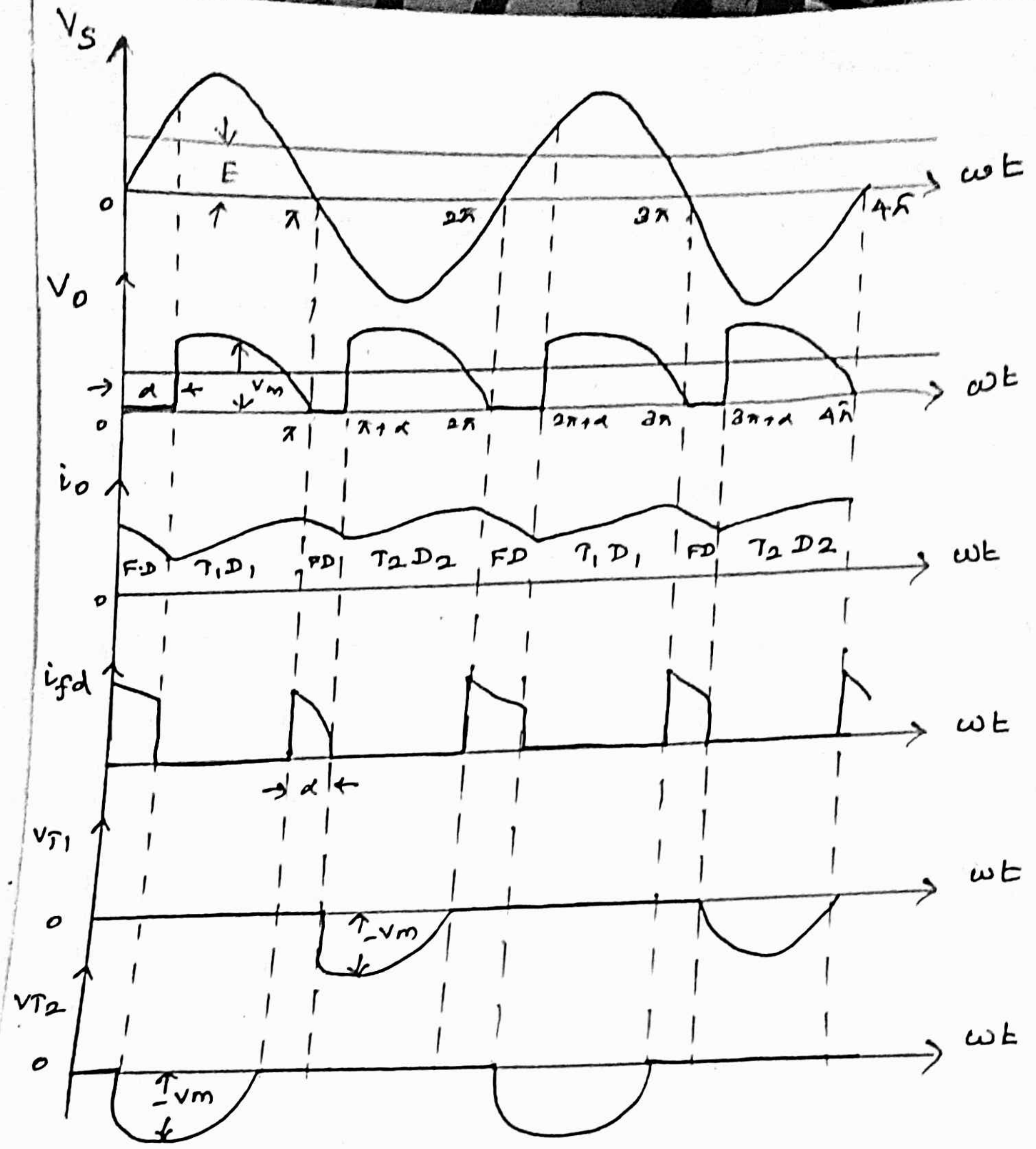
$$= \frac{V_m}{\pi} \left[\cos\alpha - \cos\beta \right] + E(\pi+\alpha-\beta].$$

~~where $\gamma = \beta - \alpha$ = conduction angle~~.

$$\cancel{= \frac{V_m}{\pi} \left[\cos\alpha - \cos\beta \right] + E}$$

<u>single phase semiconverter</u> :-





circuit operation :

T_1 is triggered at $\omega t = \alpha$, $V_m \sin\alpha > E$.

load gets connected to source through $T_1 D_1$,

$\omega t = \alpha$ to π, load current i_o flows through RLE, D_1, source and T_1.

$\omega t = \pi +$, F.D gets forward biased and starts conducting.

$\omega t = \pi + \alpha$, T_2 will triggered, F.D reverse biased.

From $\omega t = \alpha$ to π, T_1, D_1 conducts, $i_s = +ve$.

$\omega t = \pi + \alpha$ to 2π, T_2, D_2 conducts $i_s = -ve$.

$\omega t = \pi$ to $\pi + \alpha$, → freewheeling period.

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \cdot \sin\omega t \cdot d(\omega t).$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right].$$

$$= \frac{V_m}{\pi}\left[1 + \cos\alpha\right].$$

$$V_{0,r.m.s} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t \cdot d(\omega t)\right]^{1/2}.$$

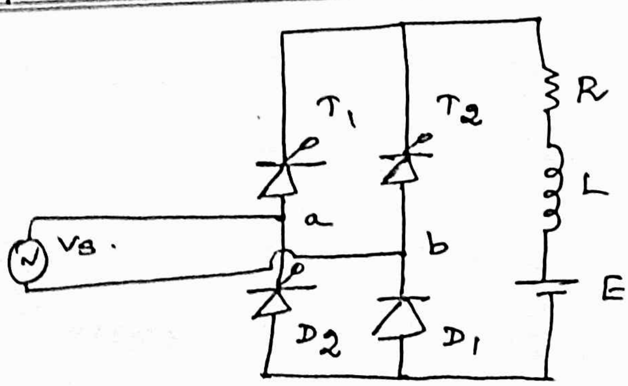
$$= \frac{V_m}{\sqrt{\pi}}\left[\int_{\alpha}^{\pi}\frac{1-\cos 2\omega t}{2}\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}\right]^{1/2}$$

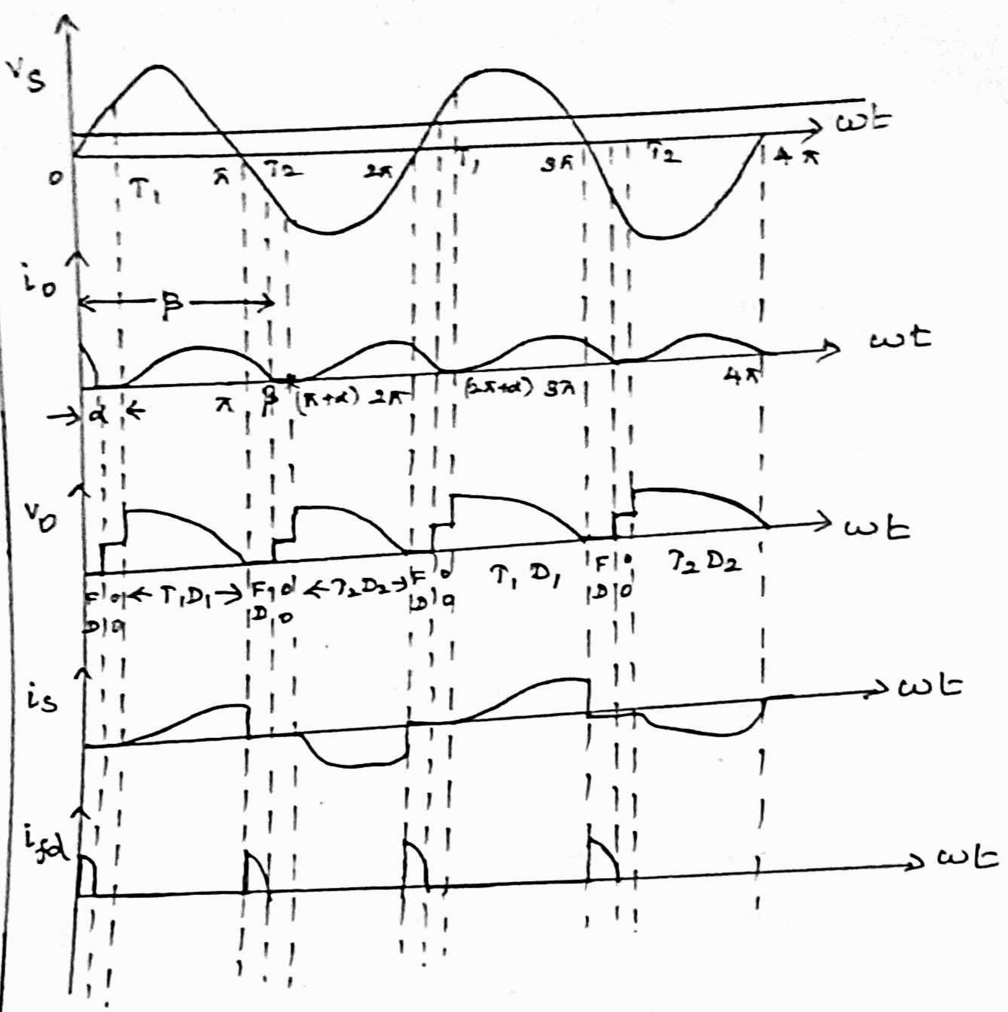
$$= \frac{V_m}{\sqrt{2\pi}}\left[\pi - \frac{\sin 2\pi}{2} - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}.$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}.$$

1 φ semiconverter with Discontinuous current :-



V_s



<u>conduction period</u> :

$\alpha < \omega t < \pi$, T_1, D_1 conduct, $V_o = V_s$.

$\pi + \alpha < \omega t < 2\pi$, T_2, D_2 conduct, $V_o = V_s$.

<u>Freewheeling period</u> :

$\pi < \omega t < \beta$, FD conducts, $i_{fd} = i_o$, $V_o = 0$.

$2\pi < \omega t < \pi + \beta$, FD conducts, $i_{fd} = i_o$, $V_o = 0$.

<u>Idle period</u> :-

$\beta < \omega t < \pi + \alpha$, no circuit component conducts.

$i_o = 0$, $V_o = E$.

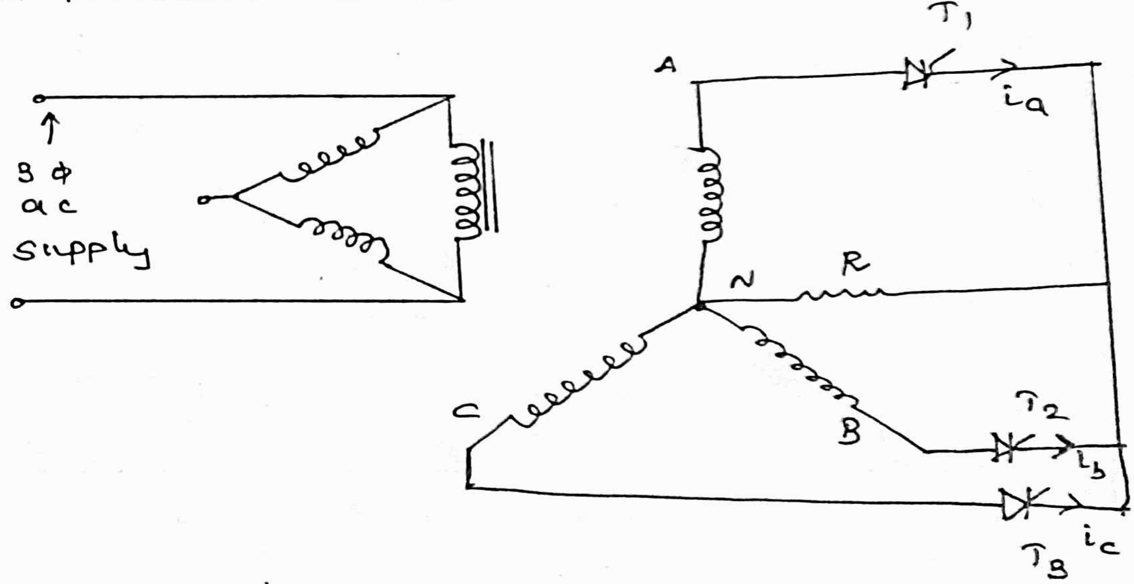
Average output voltage $V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \cdot \sin\omega t \cdot d(\omega t) + E(\pi + \alpha - \beta)$.

$= \frac{V_m}{\pi}(1 + \cos\alpha) + E(\pi + \alpha - \beta)$.

Three phase Controlled converters

The converter operating from a 1ϕ supply produces high a.c ripple voltage at its d.c terminals. smoothing reactor is necessary to smoothen the output voltage and reduce the possibility of discontinuous operation. Higher the pulse number, smoother is the output voltage. High voltages are suitably stepped down using Transformers. These transformers are delta connected on primary side and star connected on the secondary side.

Three phase Half wave controlled Rectifier with Resistive load :- [Mid point configuration]



circuit operation :-

No SCR can be triggered before a phase angle of 30°, because it remains reverse biased by the other conducting phase.

phase A and phase C are equally positive with respect to the neutral.

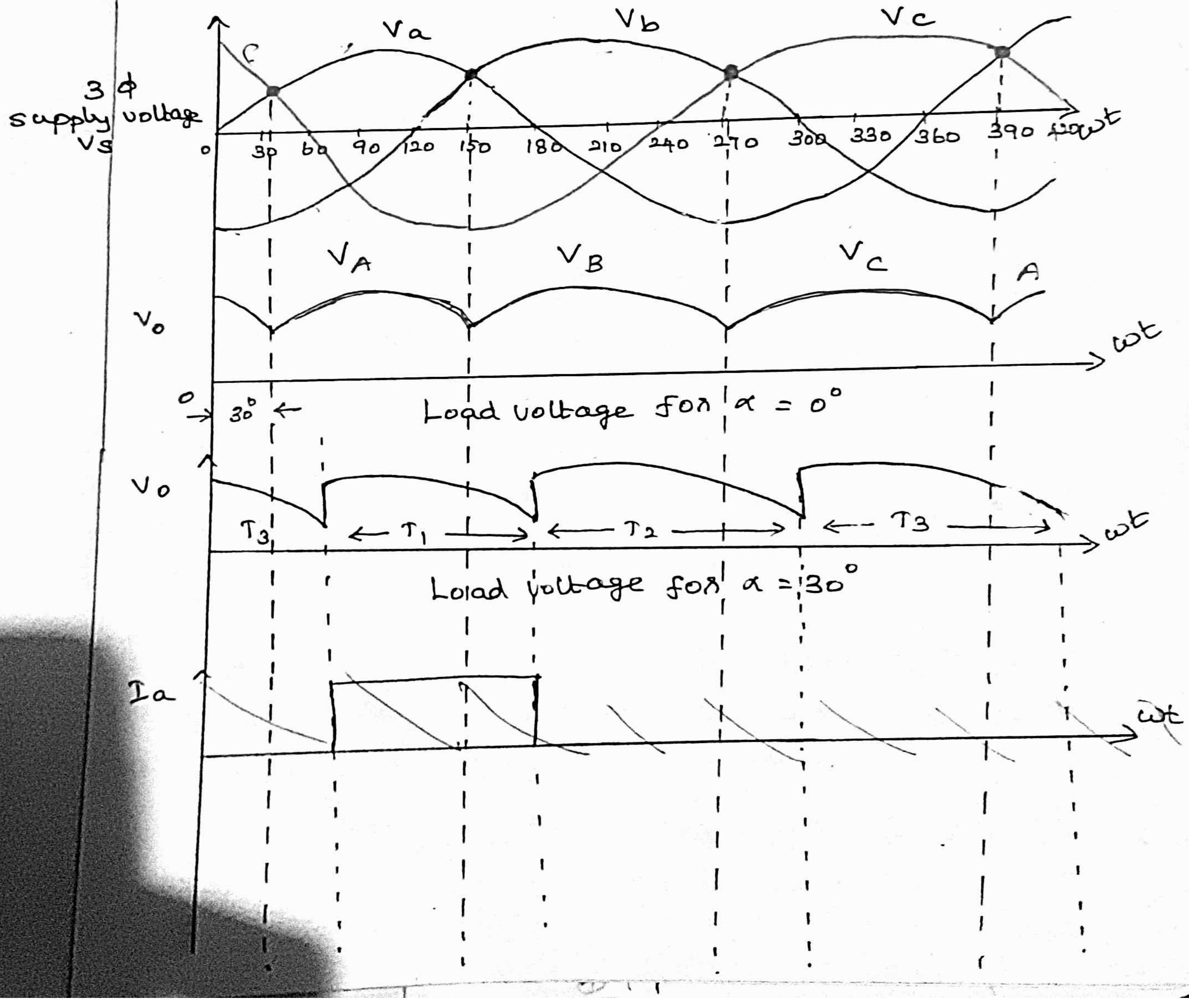
SCR T_1 connected to phase A cannot be triggered below an angle of 30°, since it is already reverse-biased by the already conducting SCR T_3.

Minimum firing angle is $\pi/6$.

T_1 conducts from $\omega t = 30^\circ$ to $\omega t = 150^\circ$.

T_2 conducts from $\omega t = 150^\circ$ to $\omega t = 270^\circ$.

T_3 conducts from $\omega t = 270^\circ$ to $\omega t = 390^\circ$.



The 3φ Half wave converter combines three single phase half wave controlled rectifiers.

Thyristor T_1 in series with one of the supply phase windings a-n acts as one half wave controlled rectifier.

second thyristor T_2 in series with supply phase windings b-n acts as second half wave controlled rectifier.

Third thyristor T_3 in series with supply phase windings c-n acts as third half wave controlled rectifier.

when thyristor T_1 is triggered at $\omega t = \pi/6 + \alpha$, Van appears across load, and T_1 conducts.

when thyristor T_2 is triggered at $\omega t = 5\pi/6 + \alpha$, T_1 becomes reverse biased and turns off, Vbn appears across load, and T_2 conducts.

when thyristor T_3 is triggered, $\omega t = 3\pi/2 + \alpha$, Vcn appears across load, T_3 conducts.

$$V_0 = 3 \times \frac{1}{2\pi} \int_{\alpha + \pi/6}^{5\pi/6 + \alpha} V_m \sin \omega t \cdot d(\omega t).$$

$$= \frac{V_m \times 3}{2\pi} \Big[-\cos \omega t \Big]_{\pi/6 + \alpha}^{5\pi/6 + \alpha}.$$

$$= \frac{V_m \times 3}{2\pi} \left[-\cos\left(\frac{5\pi}{6} + \alpha\right) + \cos\left(\frac{\pi}{6} + \alpha\right) \right]$$

$$= \frac{3V_m}{2\pi} \left[-\left[\cos\frac{5\pi}{6} \cos\alpha + \sin\frac{5\pi}{6} \sin\alpha\right] + \left[\cos\frac{\pi}{6} \cos\alpha + \sin\frac{\pi}{6} \sin\alpha\right] \right].$$

$$= \frac{3V_m}{2\pi}\Big[+ 0.866 \cos\alpha - 0.5 \sin\alpha + 0.866 \cos\alpha + 0.5 \sin\alpha \Big].$$

60.
$\frac{2\times180}{3}$

$$= \frac{3\,V_m}{2\pi}\sqrt{3}\cos\alpha .$$

$$V_o = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha .$$

Rms load voltage :

$$E_{rms} = \left[\frac{3}{2\pi}\int_{\alpha+30^\circ}^{\alpha+150^\circ} V_m^2 \sin^2\omega t \; d(\omega t)\right]^{1/2}$$

$$= V_m\sqrt{\frac{3}{2\pi}}\left[\int_{\alpha+30^\circ}^{\alpha+150^\circ} \frac{1-\cos 2\omega t}{2}\, d\omega t\right]^{1/2}$$

$$= V_m\sqrt{\frac{3}{2\pi}}\times\frac{1}{\sqrt{2}}\left[\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha+30^\circ}^{\alpha+150^\circ}\right]^{1/2}$$

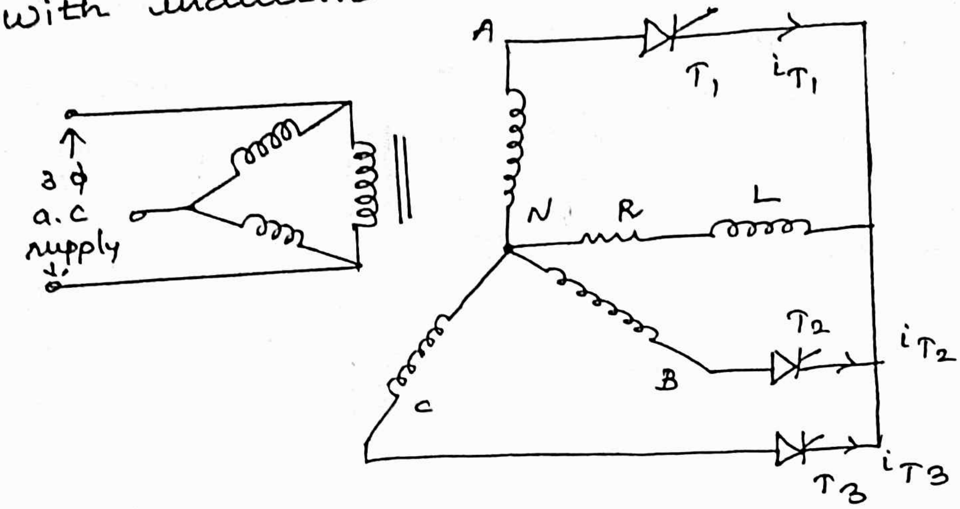
$$= \frac{V_m}{2}\sqrt{\frac{3}{\pi}}\left[\alpha+150 - \frac{\sin 2(\alpha+150^\circ)}{2} - [\alpha+30] + \frac{\sin 2(\alpha+30^\circ)}{2}\right]^{1/2}$$

$$= \frac{V_m}{2}\sqrt{\frac{3}{\pi}}\left[120^\circ - \frac{\sin 2\alpha \cos 300^\circ}{2} - \frac{\sin 300^\circ \cos 2\alpha}{2} + \frac{\sin 2\alpha \cos 60^\circ}{2} + \frac{\sin 60^\circ \cos 2\alpha}{2}\right]^{1/2}.$$

$$= \frac{V_m}{2}\sqrt{\frac{3}{\pi}}\left[120^\circ - \frac{0.5\,.\sin 2\alpha}{2} - \frac{0.\cancel{866}\,0.5\cos 2\alpha}{2} + \frac{0.5\sin 2\alpha}{2} + \frac{0.866\cos 2\alpha}{2}\right]^{1/2}.$$

$$= \frac{V_m}{2}\sqrt{\frac{3}{\pi}}\left[\frac{2\pi}{3} + \frac{0.366\cos 2\alpha}{2}\right]^{1/2}.$$

Three phase Half-wave controlled Rectifier with inductive load (R-L) :



circuit operation :-

Let the firing angle be say 45°,

T_1 conducts from $30^\circ + \alpha$ to $150^\circ + \alpha$.

T_2 conducts from $150^\circ + \alpha$ to $270^\circ + \alpha$

T_3 conducts from $270^\circ + \alpha$ to $390^\circ + \alpha$.

SCR conducts for 120°.

At $\omega t = \pi$, phase voltage V_a is zero, but i_a is not zero, because of RL load. Therefore T_1 continue conducting beyond $\omega t = \pi$.

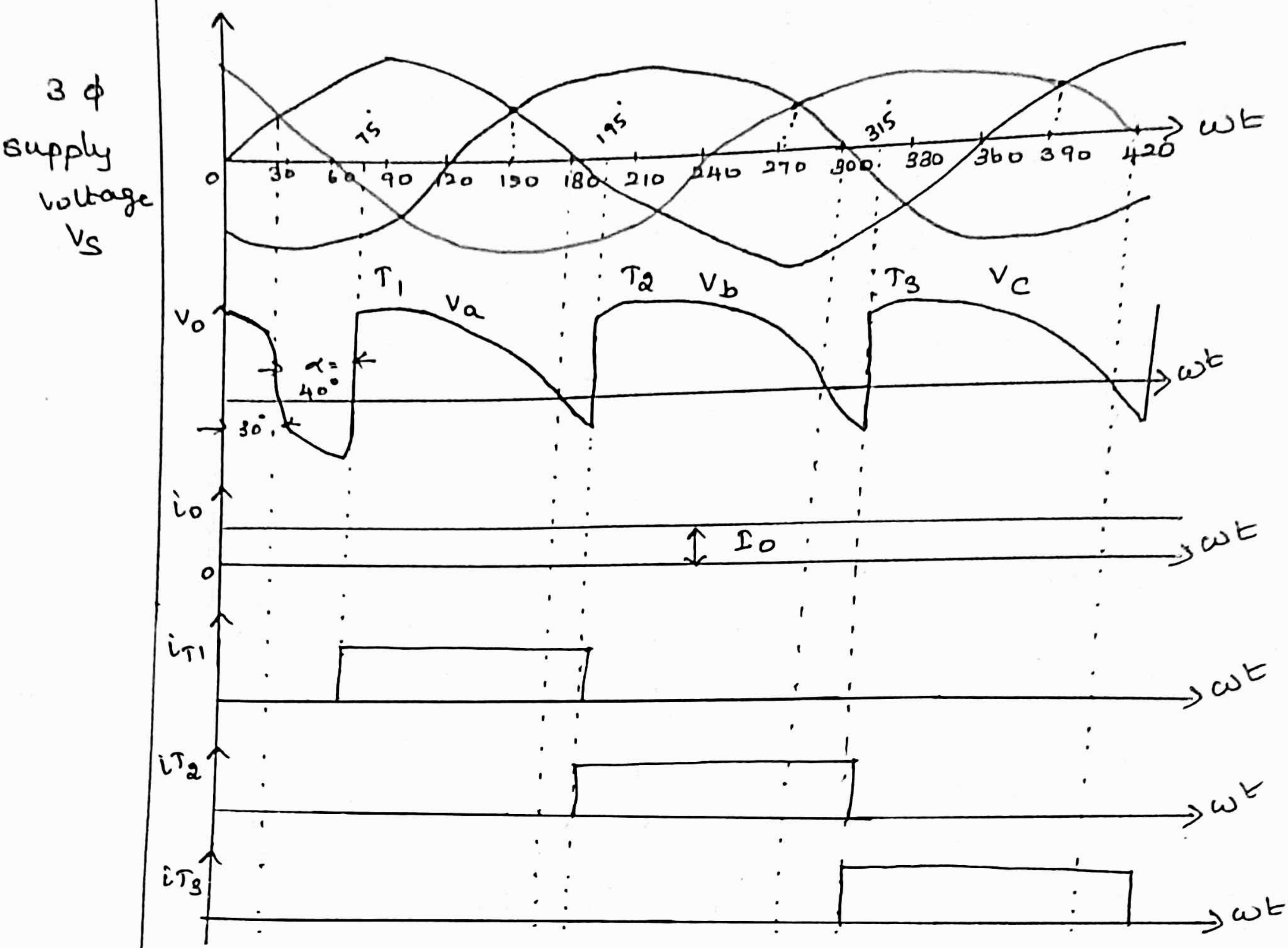
when T_1 is on, $V_{T1} = V_a - V_a = 0$, $\omega t = 75^\circ$ to 195°.

when T_2 is on, $V_{T1} = V_a - V_b$, $\omega t = 195^\circ$ to 315°.

when T_3 is on, $V_{T1} = V_a - V_c$, $\omega t = 315^\circ$ to 435°.

T_2 turned on at $\omega t = 195^\circ$,

$$V_{T1} = V_a - V_b = V_{mp} \sin 195^\circ - V_{mp} \sin 75^\circ$$
$$= -0.25 V_m - 0.96 V_m$$
$$= -1.225 V_m.$$

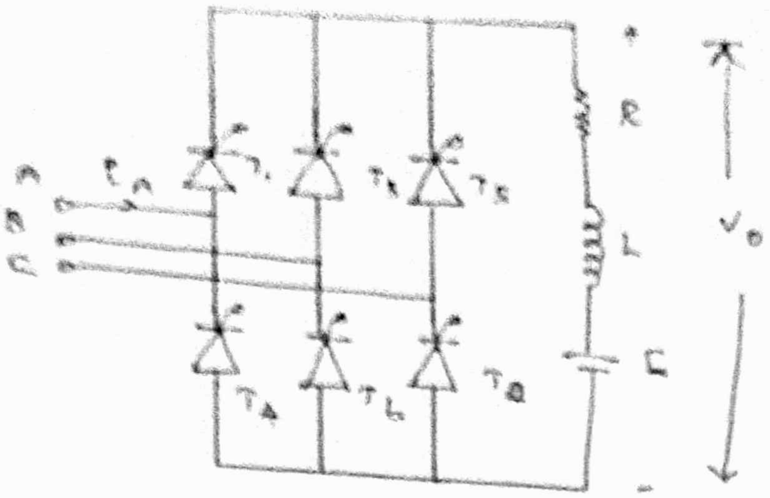


$$V_o \text{ (or) } V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin \omega t \cdot d(\omega t).$$

$$V_o = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha.$$

$$V_{rms} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m^2 \sin^2 \omega t \; d(\omega t)\right]^{1/2}.$$

Three Phase Full converters :- (6 pulse converter).



3ϕ ac to dc converter for firing angle delay $0^\circ < \alpha \leq 90^\circ$.

3ϕ line commutated inverter for $90^\circ < \alpha < 180^\circ$.

1, 3, 5 → Positive group of thyristor.

2, 4, 6 → Negative group of thyristor.

for $\alpha = 0^\circ$,

T_1 is triggered at $\omega t = 30^\circ$.

T_2 is triggered at $\omega t = 90^\circ$.

T_3 is triggered at $\omega t = 150^\circ$ and so on.

for $\alpha = 60^\circ$;

T_1 is triggered at $\omega t = 30^\circ + 60^\circ = 90^\circ$.

T_2 is triggered at $\omega t = 90^\circ + 60^\circ = 150^\circ$.

T_3 is triggered at $\omega t = 150^\circ + 60^\circ = 210^\circ$

and so on.

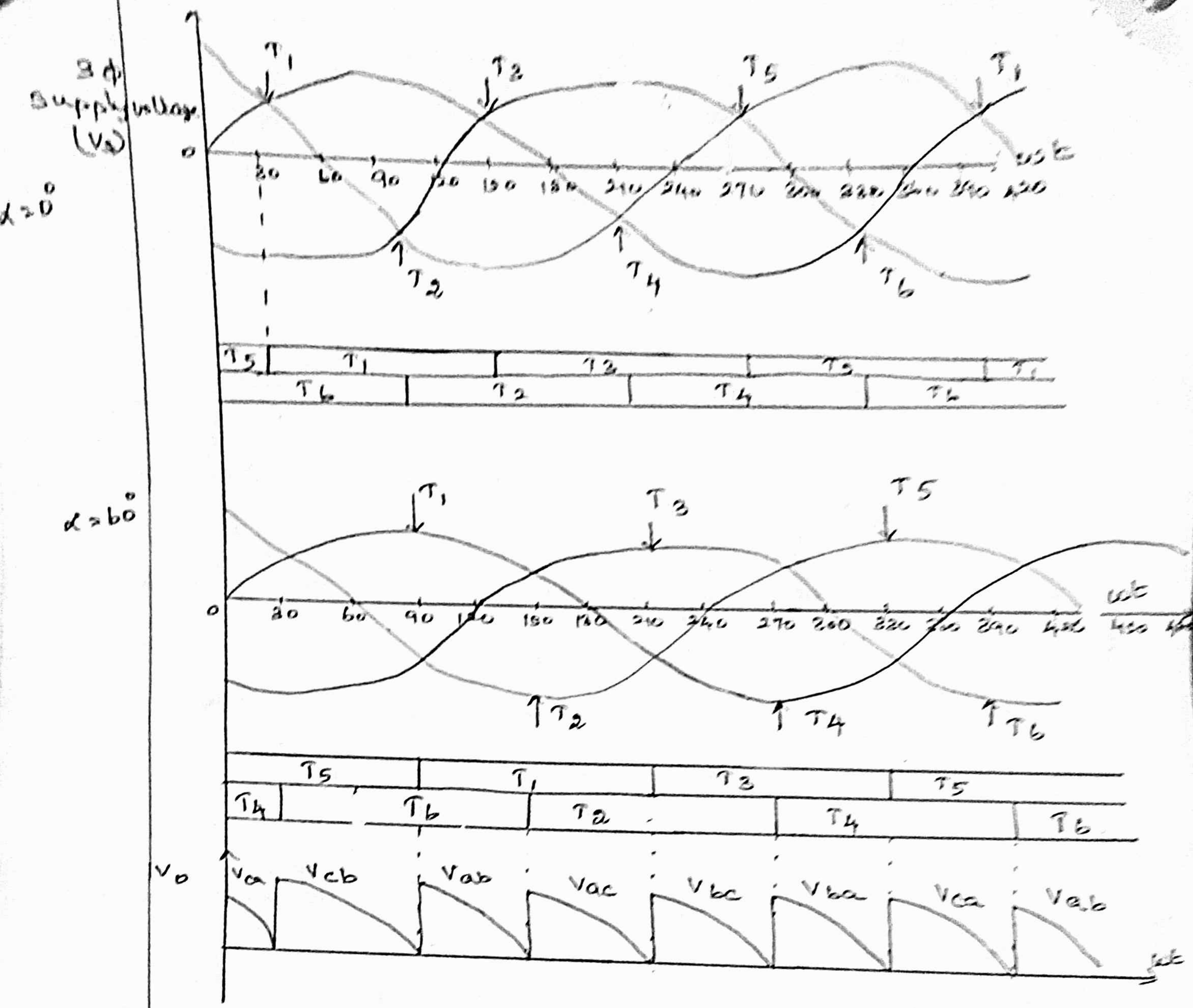
Each SCR is conducts for 120°.

+ve group of SCRs are fired at an interval of 120°

$|||^{ly}$ -ve group of SCRs are fired at interval of 120°

SCR from both the groups are fired at interval [illegible]

commutation occurs every 60°.



Line to neutral voltages are

$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

line to line voltages are

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3}\, V_m \sin(\omega t + \pi/6)$$

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\, V_m \sin(\omega t - \pi/2)$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}\, V_m \sin(\omega t + \pi/2).$$

average output voltage is found from,

$$V_o = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\pi/2+\alpha} V_{ab}\, d(\omega t).$$

$\frac{\pi}{2}+\frac{\pi}{6}$

$\frac{4\pi}{6}$

$$= \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} \sqrt{3}.\, V_m \sin(\omega t + \pi/6).\, d(\omega t).$$

$$= \frac{\sqrt{3}.\, 3V_m}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} \sin(\omega t + \pi/6)\, d(\omega t).$$

$\frac{\pi}{6} + \pi/6 = 2\pi/6 = \pi/3$

$$= \frac{\sqrt{3}.\, 3V_m}{\pi} \int_{\pi/3+\alpha}^{2\pi/3+\alpha} \sin(\omega t).\, d(\omega t).$$

$\frac{\pi}{2} + \pi/6 = \frac{3\pi+\pi}{6}$

$= \frac{4\pi}{6}$

$= 2\pi/3$

$$= \frac{\sqrt{3}.\, 3V_m}{\pi} \left[-\cos \omega t\right]_{\pi/3+\alpha}^{2\pi/3+\alpha}.$$

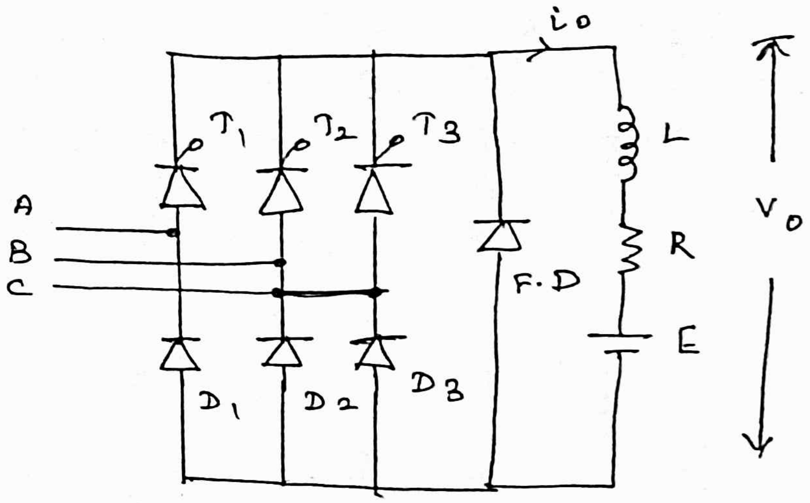
$$= \frac{\sqrt{3}.\, 3V_m}{\pi} \left[\cos(\pi/3+\alpha) - \cos(2\pi/3+\alpha)\right]$$

$$= \frac{\sqrt{3}.\, 3.\, V_m}{\pi} (\cos \pi/3 \cos \alpha - \sin \pi/3 \sin \alpha) - (\cos 2\pi/3 \cos \alpha - \sin 2\pi/3 \sin \alpha).$$

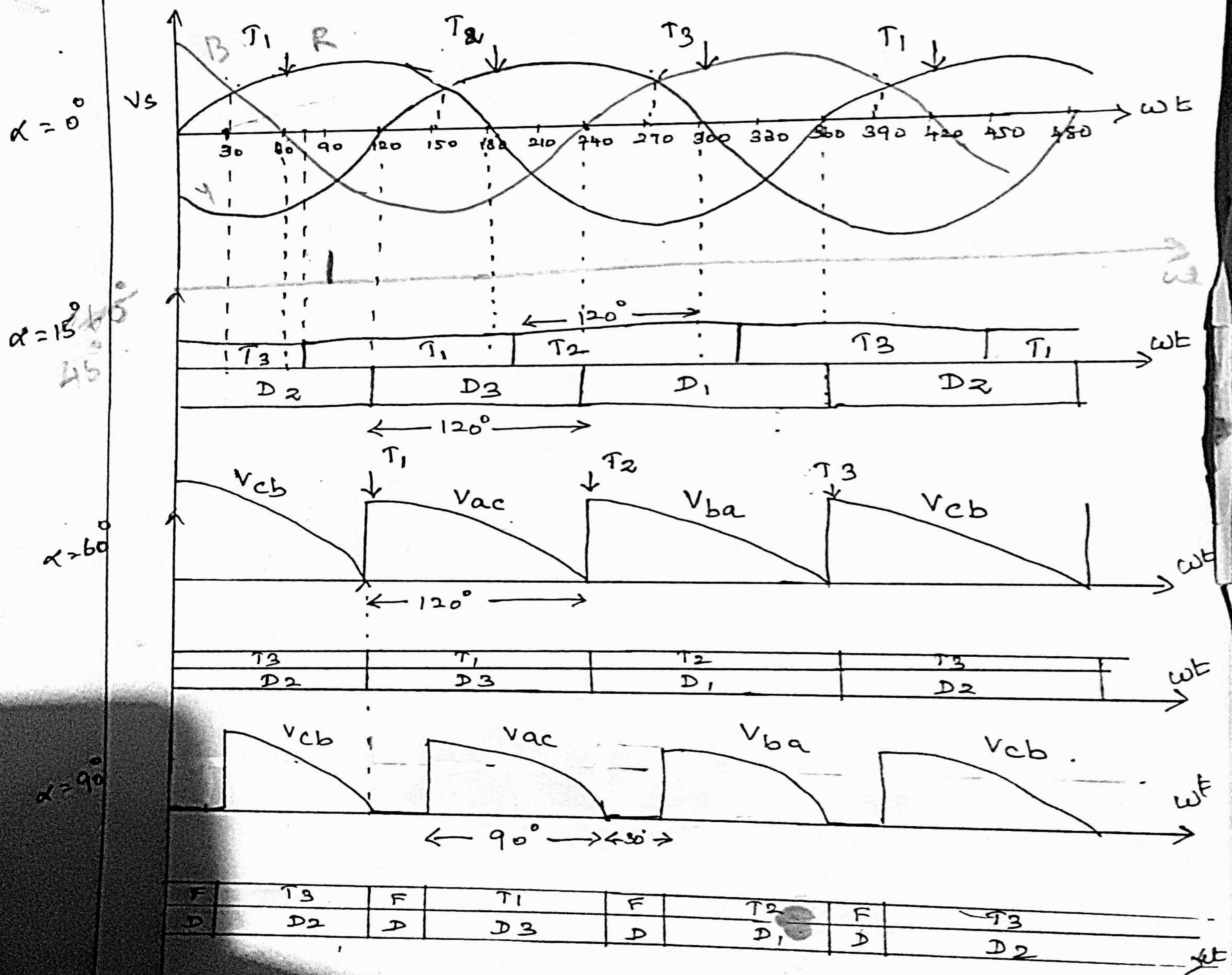
$= (0.5\cos\alpha - 0.866\sin\alpha + 0.5\cos\alpha + 0.866\sin\alpha$

$$V_o = \frac{\sqrt{3}.\, 3V_m}{\pi} \cos\alpha.$$

Three-phase semiconverter :-



$\frac{7\pi}{6}$ 30
$\frac{7\times180}{4}$
210°



3ϕ semiconverter are used in industrial applications up to the 120 kW level. The delay angle α can be varied from 0 to π. During the period $\frac{\pi}{6} \le \omega t < 7\pi/6$ T_1 is forward biased.

T_1 is fired at $\omega t = (\pi/6 + \alpha)$

T_1, D_3 conducts. V_{ac} appears across load.

At $\omega t = 7\pi/6$, V_{ac} starts to negative.
Freewheeling diode D_m conducts.

3 line-neutral voltages as follows:

$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$V_{ac} = V_{an} - V_{cn}$$

$$= \sqrt{3}\, V_m \sin(\omega t - \pi/6).$$

$$V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} V_{ac} \cdot d(\omega t)$$

$$= \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3} \cdot V_m \sin(\omega t - \pi/6) \cdot d(\omega t).$$

$$= \frac{3\sqrt{3}\, V_m}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} \sin(\omega t - \pi/6) \cdot d(\omega t).$$

$$= \frac{3\sqrt{3}\, V_m}{2\pi} (1 + \cos\alpha).$$

Three - phase semiconverter :-

Performance Parameters :-

1) Input Displacement Angle (ϕ_1) :

The angular displacement between the fundamental component of the a.c line current and the associated line to neutral voltage.

2) Input Displacement Factor ($\cos \phi_1$) :

The input displacement factor is defined as the cosine of the input displacement angle.

3) Input power factor :

It is defined as the ratio of the total mean input power to the total RMS input volt-amperes.

$$P.F = \frac{E_1 I_1 \cos \phi_1}{E_{rms} I_{rms}}$$

4) DC Voltage Ratio (r) :

It is defined as the ratio of mean d.c terminal voltage at a given firing angle α to the maximum possible d.c terminal voltage.

5) Input current Distortion Factor :

It is defined as ratio of the RMS amplitude of the fundamental component to the total RMS amplitude.

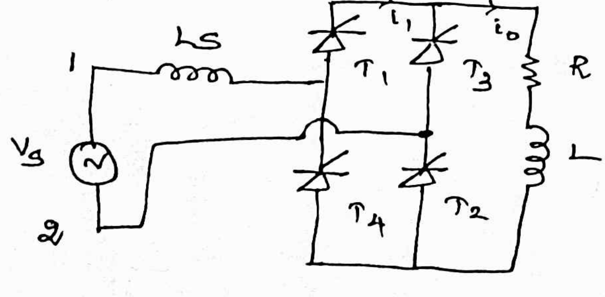
Effect of source impedance on the performance of converter :-

Incoming SCRs T_1 and T_2 are fired in a 1ϕ full converter, outgoing SCRs T_3 and T_4 get turned off. This is possible only if the voltage source has no internal impedance.

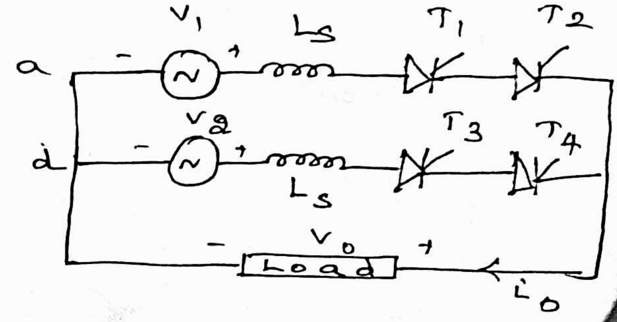
If the source impedance is resistive, there will be a voltage drop across the resistance, average voltage output of a converter gets reduced by an amount equal to $I_0 r_s$, for 1ϕ converter, and by $2 I_0 r_s$ for 3ϕ converter.

If the source impedance is inductive, it causes the outgoing and incoming SCR to conduct together. The commutation period in seconds, when outgoing and incoming SCRs are conducting together, is also known as commutation angle or overlap angle (μ) in degrees or radians.

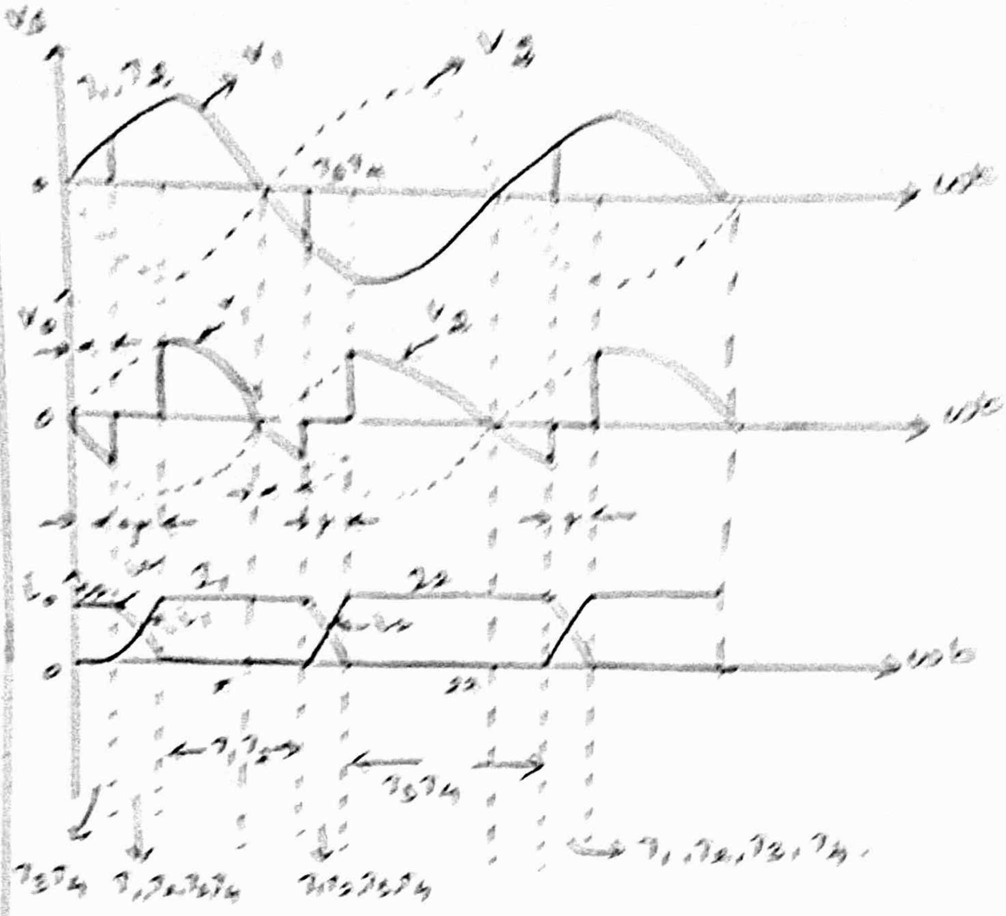
Single phase Full converter :-



1ϕ full converter with source inductance (L_s)



equivalent circuit



When terminal 1 of source voltage v_1 is positive, current i_1 flows through L_s, T_1, T_2 and load. When terminal 2 of v_2 is positive, load current i_2 flows through L_s, T_3, T_4, and load.

For the loop abcdea gives

$$v_1 - L_s \cdot \frac{di_1}{dt} = v_2 - L_s \cdot \frac{di_2}{dt}.$$

$$v_1 - v_2 = L_s \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right].$$

$v_1 = V_m \sin\omega t.$

$v_2 = -V_m \sin\omega t.$

$$L_s \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] = 2 V_m \sin\omega t.$$

Load current is assumed constant.

$$i_1 + i_2 = I_0$$

diff. w.r.to t, we get,

$$\frac{di_1}{dt} + \frac{di_2}{dt} = 0 \quad \ldots\ldots (1)$$

$$\frac{di_1}{dt} - \frac{di_2}{dt} = \frac{2V_m}{L_s} \sin \omega t \quad \ldots\ldots (2)$$

① + ② ⇒

$$\frac{di_1}{dt} = \frac{V_m}{L_s} \cdot \sin \omega t \quad \ldots\ldots (3)$$

Load current i_1 through thyristor pair T_1, T_2 builds up from zero to I_1

at $\omega t = \alpha$, $i_1 = 0$ $\quad [I_0 = I_1]$.

at $\omega t = \alpha + \mu$, $i_1 = I_0$.

From eqn ③ $\displaystyle\int_0^{I_0} \frac{di_1}{dt} \cdot dt = \int_{\alpha/\omega}^{\alpha+\mu/\omega} \frac{V_m}{L_s} \cdot \sin \omega t \cdot dt$.

$$I_0 = \frac{V_m}{\omega L_s} \Big[-\cos \omega t \Big]_{\alpha/\omega}^{\alpha+\mu/\omega}$$

$(\cos\alpha - \cos(\alpha+\mu)) \cdot \frac{I_0 \omega L_s}{I_m}$

$-\cos(\alpha+\mu) = \frac{I_0 \omega L_s}{I_m} - \cos\alpha$

$\cos(\alpha+\mu) = \cos\alpha - \frac{I_0 \omega L_s}{I_m}$

$$I_0 = \frac{V_m}{\omega L_s} \Big[\cos \alpha - \cos(\alpha + \mu) \Big] \quad \ldots\ldots (3a)$$

$$\cos\alpha - \cos(\alpha+\mu) = \frac{\omega L_s I_0}{V_m}$$

from the figure, V_0 is from $\alpha+\pi$ to $\alpha+\mu$.

$$V_{0x} = \frac{V_m}{\pi} \int_{\alpha+\mu}^{\alpha+\pi} \sin \omega t \cdot d(\omega t).$$

$$= \frac{V_m}{\pi} \Big[\cos(\alpha+\mu) - \cos(\alpha+\pi) \Big].$$

$$= \frac{V_m}{\pi} \Big[\cos \alpha + \cos(\alpha+\mu) \Big] \quad \ldots\ldots (4)$$

Vs V1 V2

Average value of ~~current~~ output voltage at no load,

$$V_0 = \frac{2V_m}{\pi} \cos\alpha$$

Maximum mean output voltage, $V_{om} = \frac{2V_m}{\pi}$

$$V_m = \frac{V_{om} \times \pi}{2}$$

$$V_{ox} = \frac{V_m}{2}$$

from eqn ④,

$$V_{ox} = \frac{\text{Maximum mean o/p voltage at no load}}{2}$$

$\cos\alpha + \cos(\alpha+\mu)$.

$$V_{ox} = \frac{V_{om}}{2}\left[\cos\alpha + \cos(\alpha+\mu)\right] \quad \ldots\ldots (5).$$

from eqn 3A.

$$\cos(\alpha+\mu) = \cos\alpha - \frac{\omega L_s}{V_m} I_0 \quad \ldots\ldots (6).$$

sub ⑥ in ④,

$$\cancel{V_{ox} = \frac{2V_m}{\pi}\left[\cos\alpha + \right.}$$

$$V_{ox} = \frac{V_{om}}{2}\left[\cos\alpha + \cos\alpha - \frac{\omega L_s}{V_m} I_0\right].$$

$$= \frac{2V_{om}}{2\pi} \cos\alpha - \frac{\omega L_s}{V_m} I_0.$$

$$V_{ox} = \frac{2V_m}{\pi} \cos\alpha - \frac{\omega L_s}{V}$$

$$V_{ox} = \frac{V_m}{\pi}\left[\cos\alpha + \cos\alpha - \frac{\omega L_s}{V_m} I_0\right].$$

$$= \frac{2V_m}{\pi} \cos\alpha - \frac{\omega L_s}{\pi V_m} I_0$$

Voltage regulation due to source inductance

$$= \frac{\omega L_s}{\pi} \times I_o \times \frac{1}{V_o \text{ at no load}}$$

$$= \frac{2\pi f L_s I_o}{\pi} \times \frac{\pi}{2 V_m \cos\alpha}$$

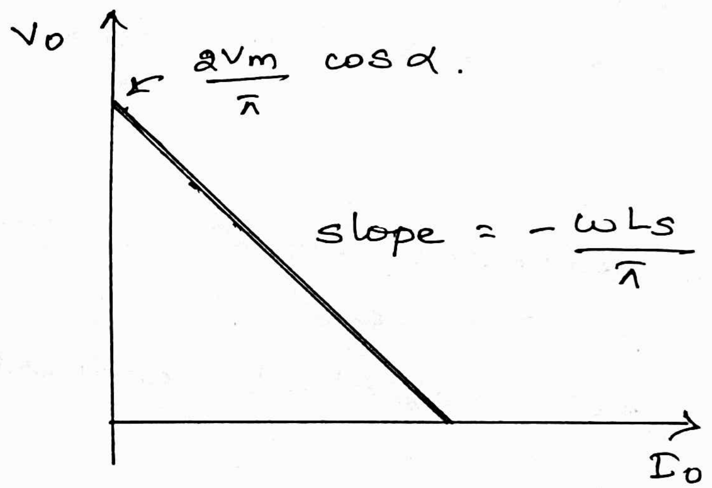
$$= \frac{\pi f L_s I_o}{V_m \cos\alpha}$$

From eqn (3a)

$$\frac{\omega L_s I_o}{\pi} = \frac{V_m}{\pi}\left[\cos\alpha - \cos(\alpha+\mu)\right].$$

For full wave diode rectifier, $\alpha = 0^\circ$,

$$\frac{\omega L_s I_o}{\pi} = \frac{V_m}{\pi}\left[1 - \cos\mu\right].$$



VS

UNIT-II (2 Marks).

① What is overlap angle (NOV/Dec 2015)

The period during which both the incoming and outgoing thyristors are conducting, is known as the overlap period. The angle for which both devices share conduction is known as the overlap angle (μ) or commutation angle.

② Mention some of the applications of converters (NOV/DEC 2015).

(i) DC motor control in steel mills, paper and textile mills employing dc motor drives.

(ii) Reactor controls

(iii) Portable hand tool drives.

(iv) AC fed traction system using dc traction motor.

③ classify the different types of controlled rectifier: (NOV/DEC 2016).

(i) 1 ϕ half wave controlled converter

(ii) 1 ϕ semi converter.

(iii) single phase full converter.

iv) 3 ϕ half wave controlled converter.

v) 3 ϕ semiconverter.

vi) 3 ϕ full converter.

④ What is the function of freewheeling diode and state its advantages : (NOV/DEC 2016)

(i) The continuous current will flow in the load after the SCR is turned off, due to the energy stored in the inductor.

(ii) The average value of the o/p voltage is same as that of with Resistive load.

(iii) The converter has a better power factor due to this freewheeling action.

⑤ What is the effect of source impedance on the performance of converter? (Apr/May 2015)

(i) To lower the mean output voltage.

(ii) To distort the output voltage and current waveforms.

(iii) To modify the performance parameters of the converter.

⑥ Compare half controlled rectifier and Full controlled rectifier :

Half controlled rectifier	Full controlled Rectifier
i) It produces only one pulse during the cycle.	i) It produces two pulses during the cycle.
(ii) consists of both thyristor and diodes. It rectify only half of the wave.	consists of two thyristor and rectify both part of waveform.

(7) What is meant by forced commutation :

In case of d.c circuits, for switching off thyristors, the forward current of the thyristor is forced to zero, by an additional circuit called commutation circuit. This is called forced commutation

(8) What is meant by firing angle (or) delay angle ?

The angle between the zero crossing of the input voltage and the instant the thyristor is fired.

(9) What are the advantages of 6 pulse converter ?

(i) commutation is made simple.

(ii) Distortion on the ac side is reduced due to the reduction in lower order harmonics.

(10) What is meant by input power factor in controlled Rectifier ?

It is defined as the ratio of the total mean input power to the total RMS input volt-amperes.

(11) Define pulse number ?

Pulse number is defined as the number of pulses in the dc output voltage within one time period of the ac source voltage. For e.x 1 ϕ half controlled rectifiers produce only one pulse of load current during one cycle of source voltage, it can be termed as 1 ϕ one pulse converters.

12) Define Input power factor :-

It is defined as the ratio of the total mean input power to the total RMS input volt - amperes.

$$p.f = \frac{E_1 I_1 \cos \phi_1}{E_{rms} I_{rms}}$$

13) what is meant by commutation of SCR and its types.

A thyristor is turned on by applying a signal to its gate - cathode circuit. Commutation means a process of turning off a thyristor.

Types :-

1) Load commutation
2) Resonant pulse commutation
3) Impulse commutation
4) External pulse commutation
5) Line commutation.

14) write the relation between firing angle and extinction angle in 1ϕ fully controlled rectifier when operating with RL load!

$\alpha \rightarrow$ firing angle.

$\beta \rightarrow$ extinction angle.

$\gamma \rightarrow$ conduction angle.

$$\boxed{(\beta - \alpha) = \gamma}$$

$\omega t = \alpha$, $V_T = V_m \sin\alpha$,

$\omega t = \beta$, $V_T = V_m \sin\beta$; $\beta > \pi$, V_T is negative at $\omega t = \beta$.

Thyristor reverse biased from $\omega t = \beta$ to 2π.

15) What is meant by phase control ?

The firing angle is defined as the angle between the zero crossing of the input voltage and the instant the thyristor is fired. The most efficient method to control the turning ON of a thyristor is achieved by varying the firing angle of SCR.

16) Define harmonic factor (or) THD of the input current.

→ The ratio of the total harmonic ~~current~~ content to the fundamental component.

$$THD = \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}}$$

17) Define Displacement factor :-

The cosine of the input displacement angle.

18) Differentiate the device turn off time from the circuit turn off time.

Circuit turn off time	Device turn off time.
1) Time duration from the start of SCR reverse biasing of outgoing SCR to the forward biasing of incoming SCR.	Due to reversal of supply the conducting SCR is turn off.
2) More value compared to device turn off time.	Lesser value.

single phase and 3ϕ AC voltage controllers - control strategy - Power factor control - Multistage sequence control - single phase and three phase cyclo converters - Introduction to Matrix converters.

1ϕ AC voltage controllers :

AC voltage controllers are semiconductor based circuits which convert fixed alternating voltage to variable alternating voltage directly without change in the frequency.

Applications :

i) Domestic and Industrial heating
ii) Transformer tap changing
iii) Lighting control
iv) speed control drives
v) starting of Induction motors :

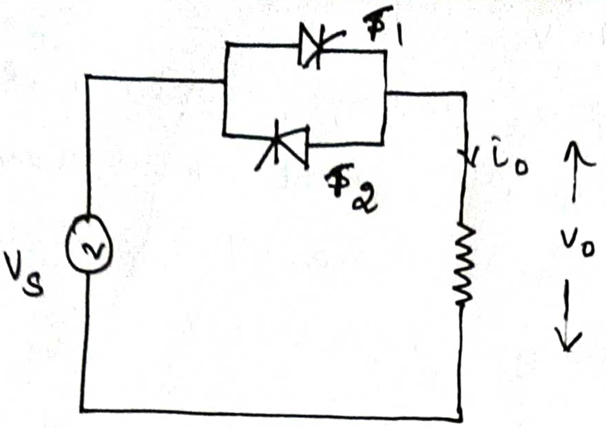
single phase AC voltage controller with Resistive load :

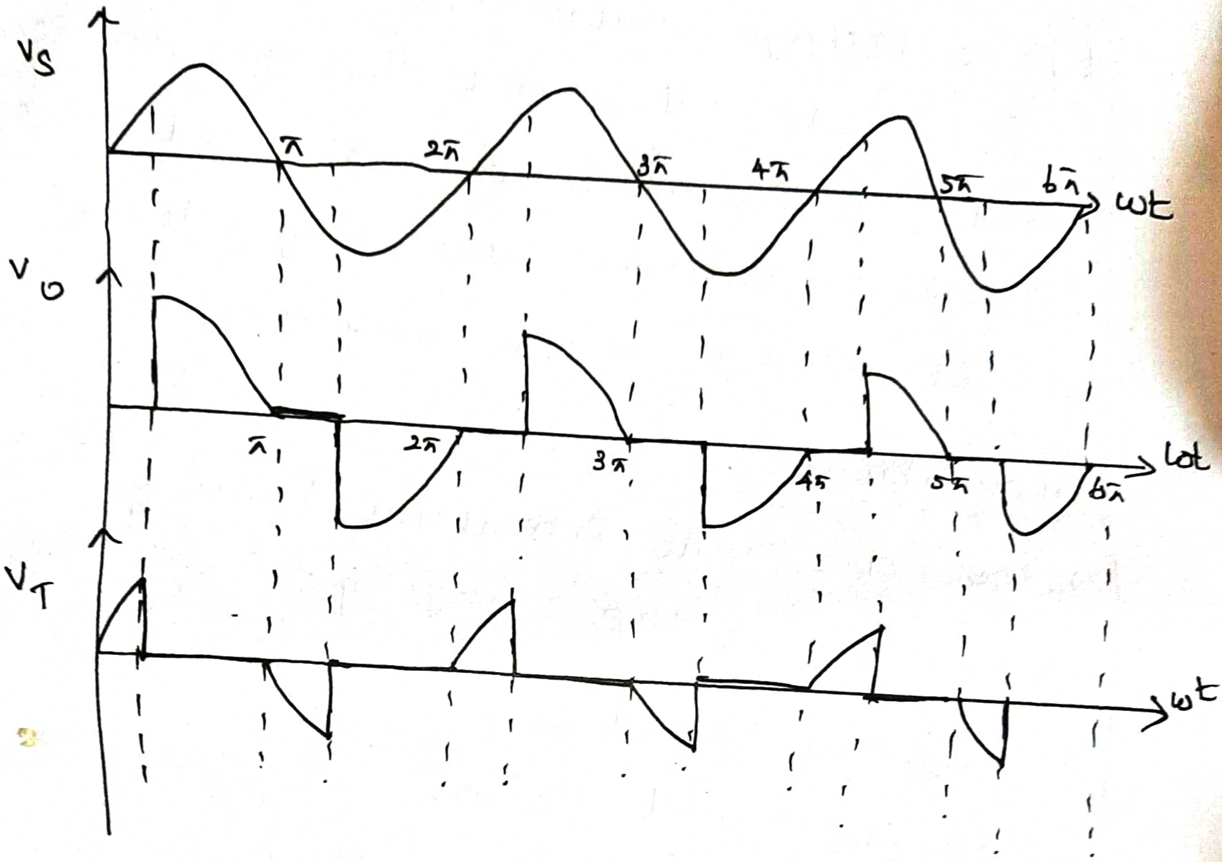
Phase control :-

It consists of two SCRs connected in antiparallel. So, it is possible to have current flow in either direction.

During +ve half cycle, T_1 is triggered at $\omega t=\alpha$, it conducts from $\omega t = \alpha$ to π,

During -ve half cycle, T_2 is triggered at $\omega t = \pi+\alpha$, it conducts from $\omega t = \pi+\alpha$ to 2π.





RMS Voltage :

$$V_r = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \cdot d\omega t}.$$

$$= \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t).\, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}\right]^{1/2}$$

$$= \frac{V_m^2}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$V_r = \frac{V_m}{\sqrt{2\pi}}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)^{1/2}.$$

RMS load current

$$I_r = \frac{V_r}{R}$$

$$= \frac{V_m}{\sqrt{2\pi}\,R}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)^{1/2}$$

Power factor of the load :-

$$= \frac{V_r^2/R}{V_s \cdot I_s}$$

$$= \frac{V_r^2/R}{V_s \times V_r/R} = \frac{V_r}{V_s}$$

$$= \frac{\frac{V_m}{\sqrt{2\pi}}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)^{1/2}}{V_m/\sqrt{2}}$$

$$P.F = \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

1 ϕ AC voltage controller with RL load :-

⇒ During +ve half cycle, $\omega t = 0$ to π, T_1 is forward biased, $\omega t = \alpha$, T_1 is triggered.

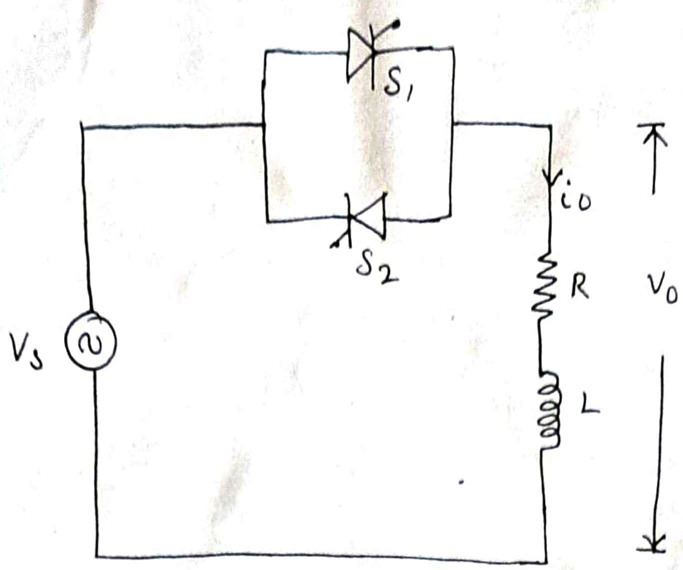
⇒ At $\omega t = \pi$, current is not zero, T_1 is reverse biased, but does not turn off because of i_o.

⇒ From β to $\pi + \alpha$, no current exist in the power circui

⇒ T_2 turned on at $(\pi + \alpha) > \beta$, $i_o = i_{T2}$.

β is called extinction Angle

$\boxed{\text{operation in page No : 6}}$ → P.T.O.



expression for load current (i_0) :-

KVL for the circuit, when T_1 conducts,

$$V_s = V_m \sin\omega t = R i_0 + L \cdot \frac{di_0}{dt} \quad \ldots \alpha < \omega t < \beta .$$

solution of this equation is,

$$i_0 = \frac{V_m}{Z} \sin(\omega t - \phi) + A \cdot e^{-(R/L)t} \quad \ldots \text{(I)}.$$

$$Z = [R^2 + \omega L^2]^{1/2} \; ; \; \phi = \tan^{-1} \omega L/R .$$

Boundary conditions are, $\omega t = \alpha$, $t = \alpha/\omega$; $i_0 = 0$.

$$0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A e^{-R\alpha/L\omega}$$

$$\boxed{A = \frac{-V_m}{Z} \sin(\alpha - \phi) e^{R\alpha/L\omega}} \quad \ldots \text{(II)}.$$

sub II in (I)

$$i_0 = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{V_m}{Z} \sin(\alpha - \phi) \cdot e^{R/L(\alpha/\omega - t)}.$$

(A)

RMS output voltage :

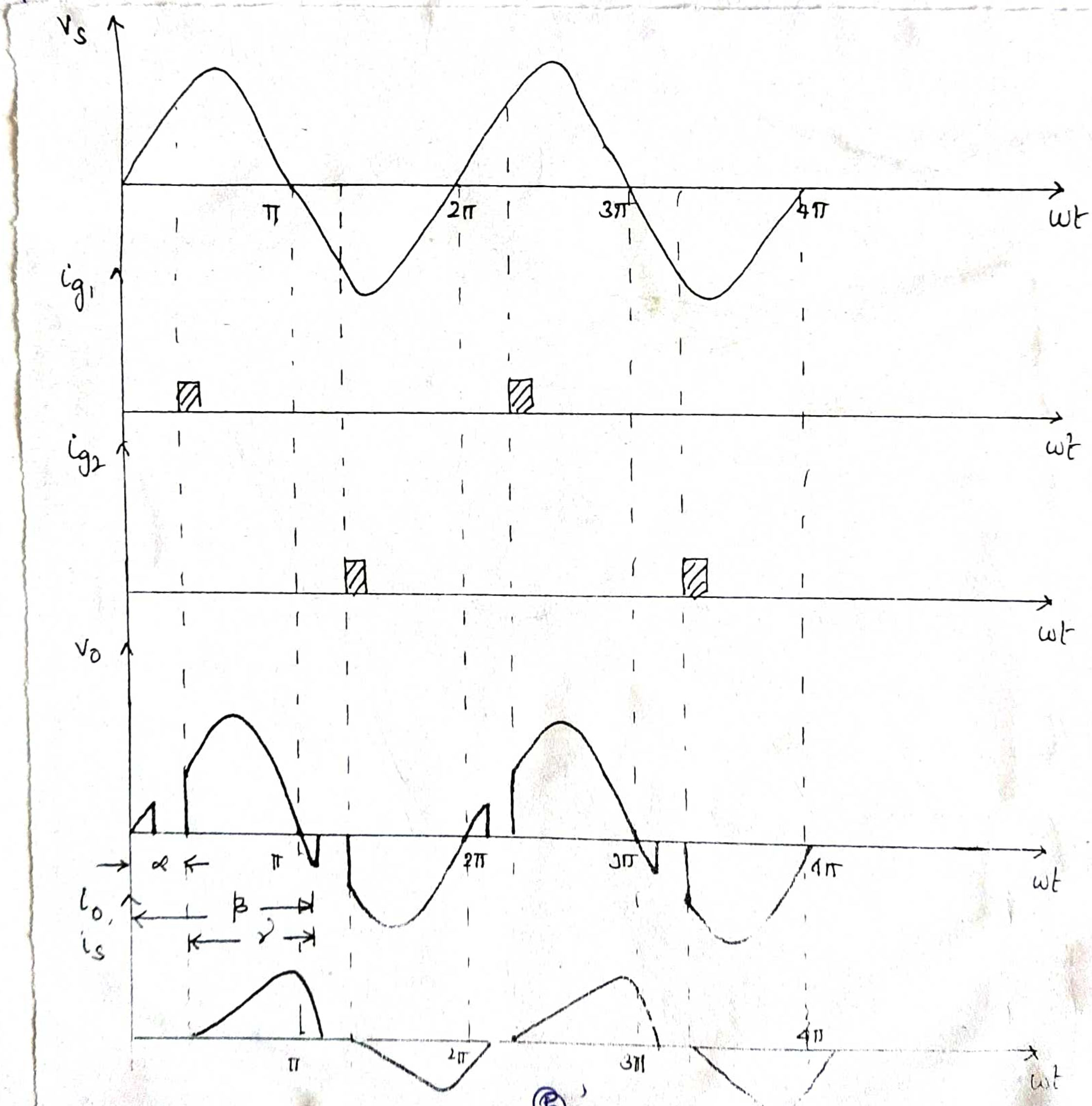
$$V_r = \left[\frac{1}{\pi}\int_{\alpha}^{\beta} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi}\int_{\alpha}^{\beta} \frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\beta}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left[\beta - \frac{\sin 2\beta}{2} - \alpha + \frac{\sin 2\alpha}{2}\right]\right]^{1/2}$$

$$\boxed{V_r = \frac{V_m}{\sqrt{2\pi}}\left[\beta - \alpha + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]^{1/2}}$$



With RL load

Principle of operation :-

⇒ During 0 to π, S_1 is forward biased.

⇒ at $\omega t = \alpha$, S_1 is triggered and i_o starts building up through the load.

⇒ at $\omega t = \pi$, load & source voltage are zero, but the current is not zero, due to the presence of inductance. So, load current extends upto β. angle β is called extinction Angle.

after π, S_1 is reverse biased but does not turn off due to i_o is not zero.

⇒ when i_o is zero, S_1 is turned off.

from β to $\pi + \alpha$, no current exists in the power circuit.

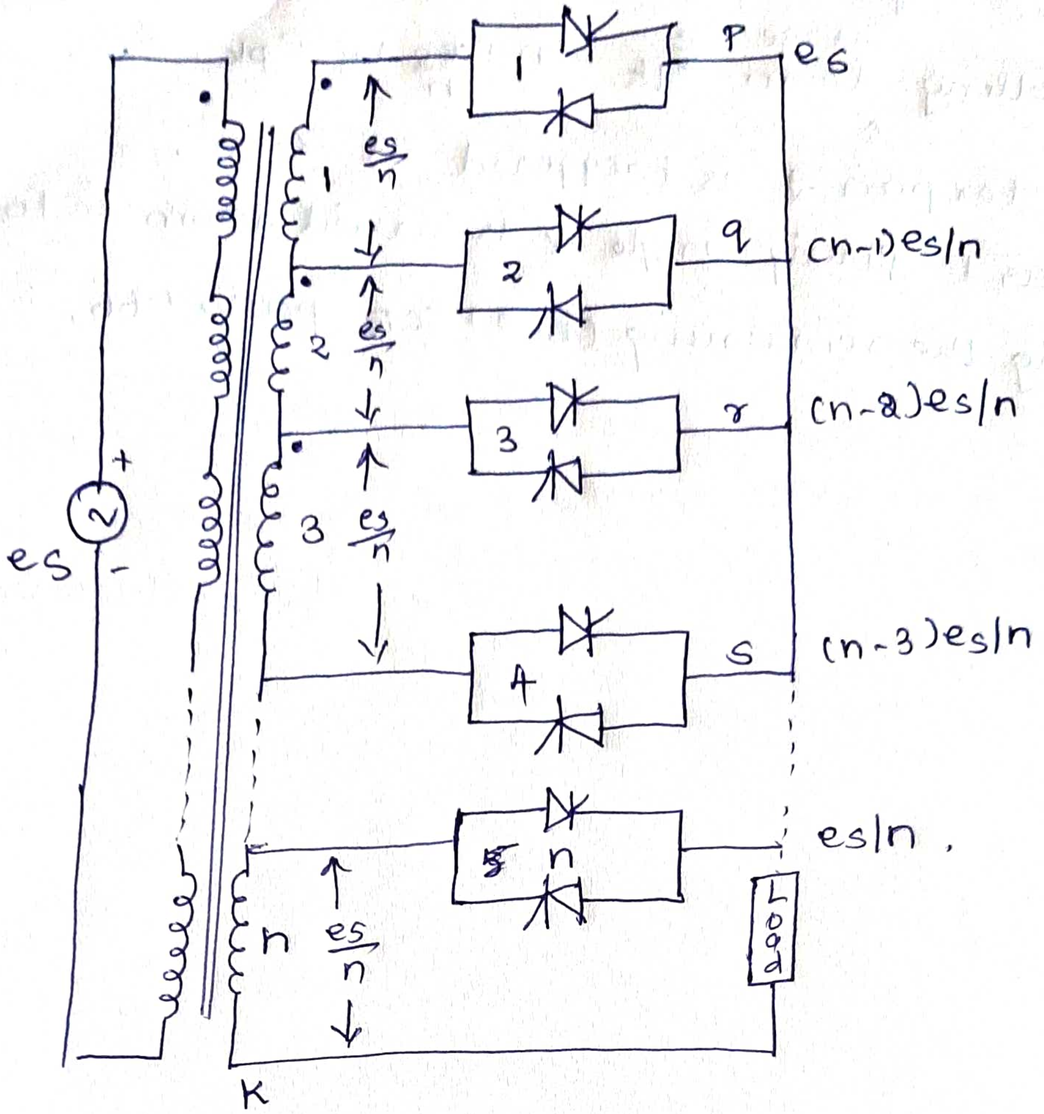
⇒ Thyristor S_2 turned ON at $\omega t = \pi + \alpha$.

at $\omega t = 2\pi$, V_s, V_o becomes zero, but i_{S2} is not zero due to inductive load.

At $\omega t = \pi + \alpha + \gamma$, $i_{S2} = 0$, S_2 is turned off.

again at $2\pi + \alpha$, S_1 is turned ON, and current start building up.

Multistage sequence Control of A.C Regulators



⇒ By using more than two stages of sequence control, it is possible to have further improvement in Power factor and reduction in harmonics.

⇒ The transformer has n secondary windings, each secondary is rated for e_s/n, $e_s \rightarrow$ source voltage.

⇒ The voltage of node P with respect to K is e_s. voltage of terminal q is $(n-1)e_s/n$.

⇒ voltage control from

$e_{sk} = (n-3)\frac{e_s}{n}$ to $e_{sk} = (n-2)\frac{e_s}{n}$ is required,

thyristor pair 4 is triggered at $\omega t = 0°$, firing angle of thyristor pair 3 is controlled from $\alpha = 0°$ to $180°$, all other thyristor pairs are kept off.

⇒ controlling voltage from $e_{qk} = (n-1)\frac{e_s}{n}$ to $e_{pk} = e_s$,

thyristor pair 2 is triggered at $\alpha = 0°$,

pair 1 firing angle α is varied from $0°$ to $180°$,

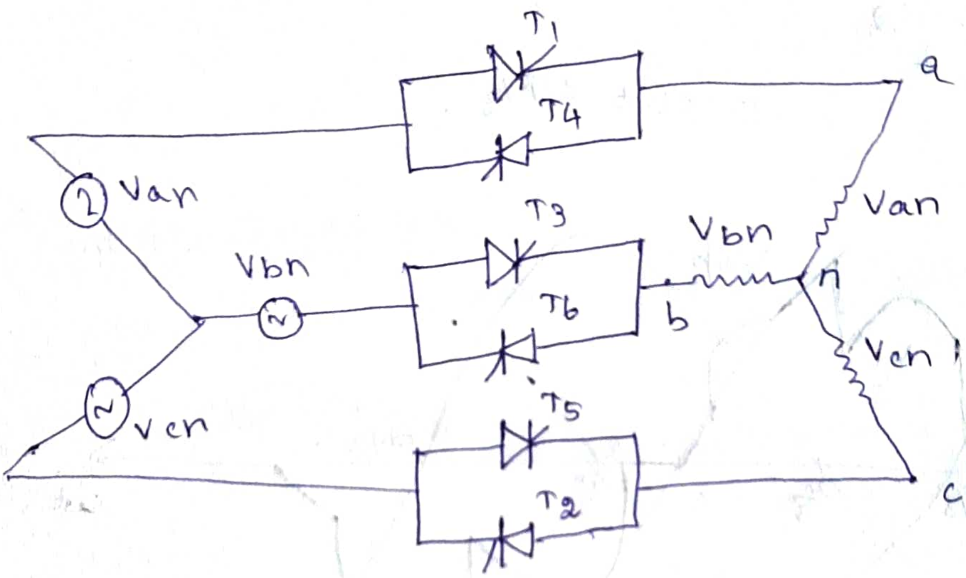
keeping the remaining $(n-2)$ SCR pairs off.

Three phase Bidirectional Delta connected controllers :-

Three Phase AC Regulators.

The 3ϕ ac full wave controller with star connected load is shown in figure. If a neutral connection is made, load current can flows provided atleast one thyristor is conducting. At high power level, neutral connection is to be avoided. Because of load triplen currents, they may flow through the phase inputs and the neutral.

Without the neutral connected, each device would conduct for $\pi/2$ in the order T_1 to T_6 at $\pi/3$ apart.



=) If the thyristor T_1 is triggered at α_1, then for a symmetrical 3ϕ load voltage, the other trigger angles are T_3 at $\alpha + \frac{2}{3}\pi$ and T_5 at $\alpha + \frac{4\pi}{3}$

For the antiparallel devices, T_4 is at $\alpha + \pi$, T_6 at $\alpha + \frac{5\pi}{3}$, T_2 at $\alpha + \frac{7\pi}{3}$.

(i) $0 \le \omega t \le \pi/3$:-

Full output occur when $\alpha = 0$. For $\alpha \le \pi/3$, 3 alternating devices conduct and one will be turned off by natural commutation. only for $\omega t \le \pi/3$, can three sequential devices be on simultaneously.

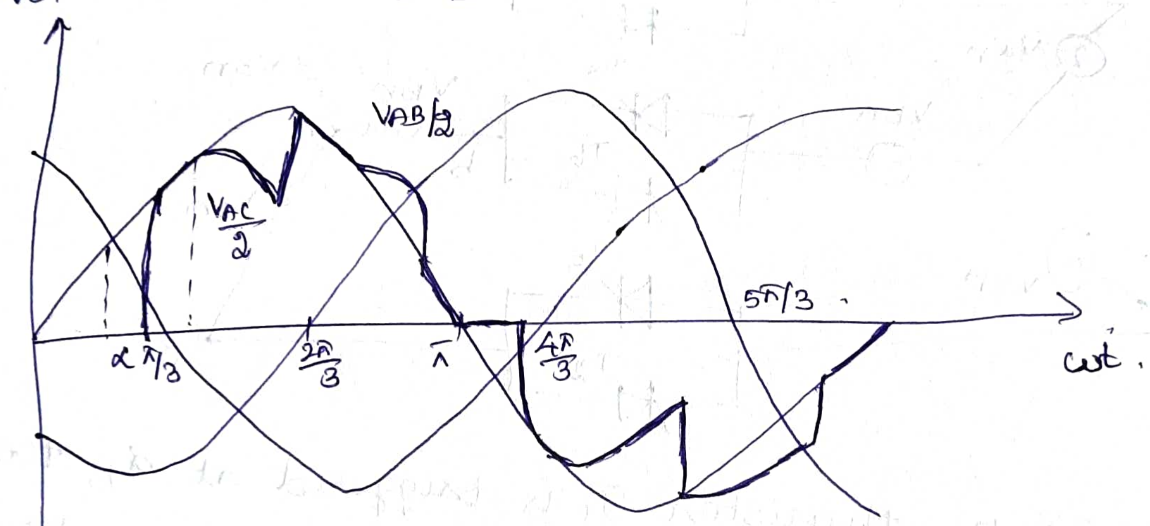
(ii) $\pi/3 \leq \omega t \leq \pi/2$

The turning ON of one device naturally commutate another conducting device and only two phase can be conducting, that is only two thyristors conduct at any time. Line to neutral voltage waveform for $\alpha = \pi/3$ & $\alpha = \pi/2$ shown in fig.

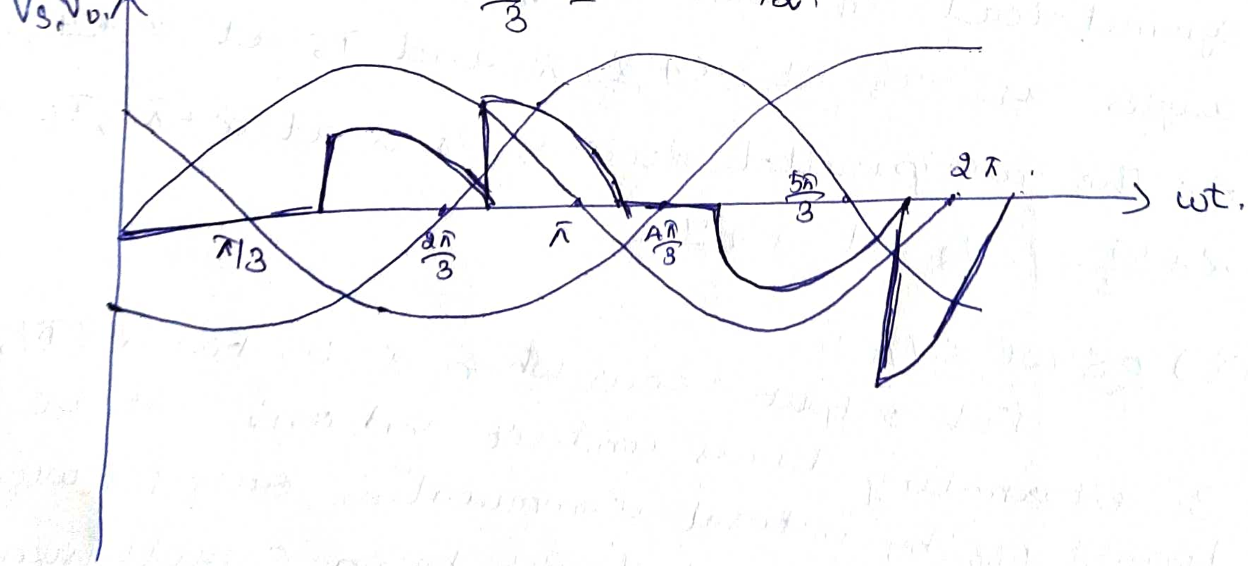
3 thyristor conduct, the voltage is of the form $\frac{V_{mL}}{\sqrt{3}} \sin \omega t$.

when 2 thyristor conduct, the voltage is of the form $\frac{V_{mL}}{2} \sin(\phi - \pi/2)$.

$0 \leq \omega t \leq \pi/3$.



$\frac{\pi}{3} \leq \omega t \leq \pi/2$.



For $\alpha = \pi/4$, the rms load voltage/phase is,

$$V_{rms} = V_{ml}\left[\frac{1}{\pi}\left(\int_{\alpha}^{\pi/3}\frac{\sin^2\omega t}{(\sqrt{3})^2}\,d\omega t + \int_{\pi/3}^{\pi/3+\alpha}\frac{\sin^2(\omega t-\pi/6)}{2^2}\,d\omega t\right.\right.$$

$$+ \int_{\pi/3+\alpha}^{2/3\,\pi}\frac{\sin^2\omega t}{(\sqrt{3})^2}\,d\omega t + \int_{2\pi/3}^{\frac{2\pi}{3}+\alpha}\frac{\sin^2(\omega t-\pi/6)}{2^2}\,d\omega t +$$

$$\left.\int_{\frac{2\pi}{3}+\alpha}^{\pi}\frac{\sin^2\omega t}{(\sqrt{3})^2}\,d\omega t\right]^{1/2}$$

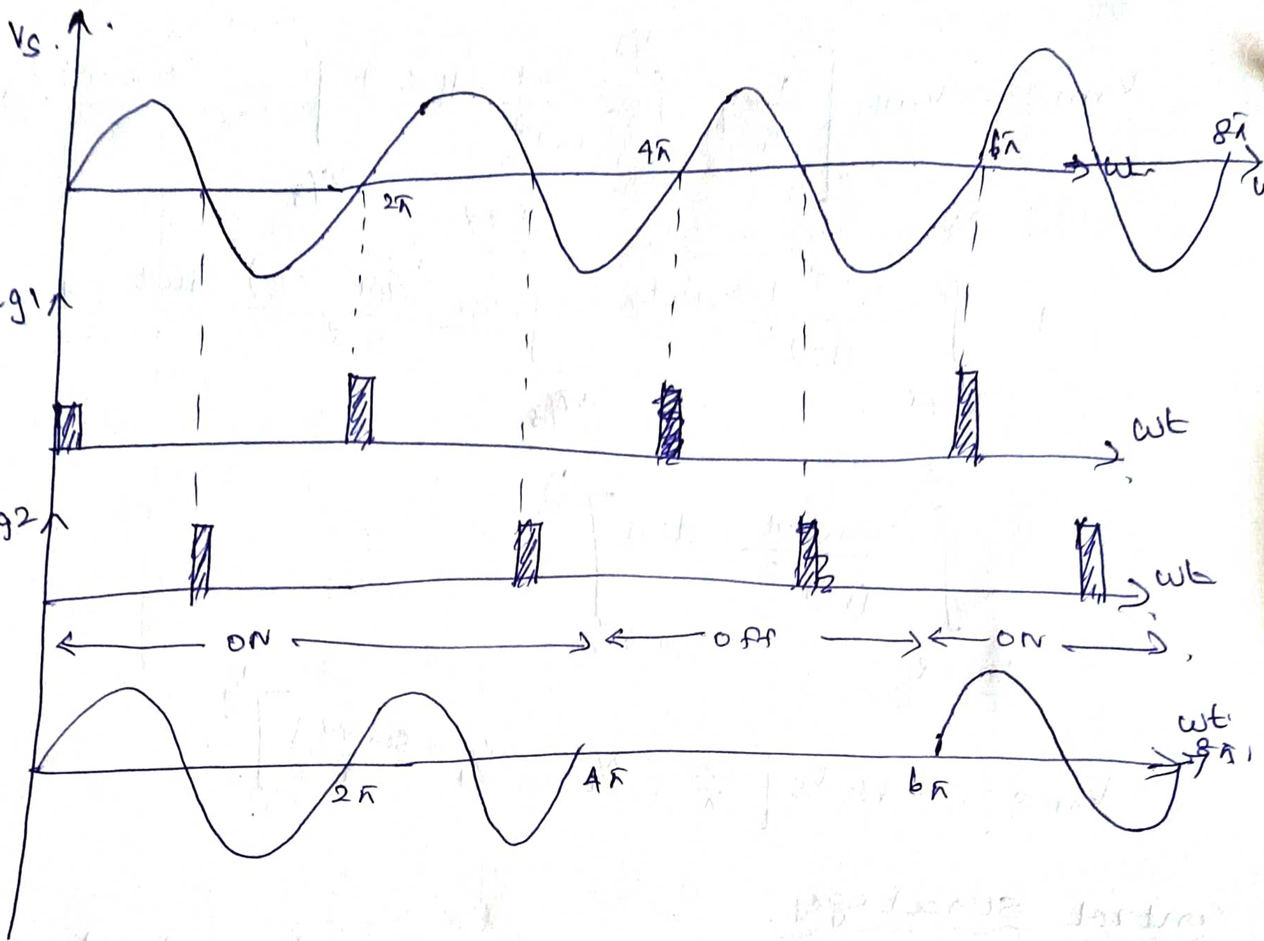
$$V_{rms} = \sqrt{6}\,V_s\left[\frac{1}{\pi}\left(\pi/6 - \alpha/4 + \frac{\sin 2\alpha}{8}\right)\right]^{1/2}$$

Control strategy :-

Power factor control : Integral cycle control.

In industry for some applications, almost no variation in speed is the control is achieved by connecting load to source for some cycles, and then disconnecting the load for some off cycles, It is called integral cycle control.

The source energies the load for n cycles. Load remains off when gate pulses are withdrawn. for m cycles. By varying the numbers of n & m cycles, power delivered to load can be regulated.



RMS load voltage

$$V_{rms} = \frac{1}{\text{Periodicity}} \int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \ (I^{st}\text{ cycle}) + \int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \ (2^{nd}\text{ cycle}) + \dots \int_0^{} V_m^2 \sin^2 \omega t \, d\omega t \ (n^{th}\text{ cycle}).$$

For n on cycle, m off cycle, periodicity $= (n+m)2\pi$.

$$V_r = \left[\frac{1}{2\pi(n+m)} \cdot n \int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t\right]^{1/2}.$$

$$= \left[\frac{nV_m^2}{4\pi(n+m)} \int_0^{2\pi} (1-\cos 2\omega t)\, d\omega t\right]^{1/2}.$$

$$= \left[\frac{n \cdot V_m^2}{4\pi(n+m)} [2\pi]\right]^{1/2}.$$

$$V_r = \frac{V_m}{\sqrt{2}} \sqrt{\frac{n}{n+m}}.$$

Take $k = \sqrt{\frac{n}{n+m}}$.

$V_r = V_s\sqrt{k}$.

RMS load current

$$I_r = \frac{V_r}{R}$$

$$= \frac{V_s\sqrt{k}}{R}$$

Power delivered to the load :

$$P = \frac{V_r^2}{R} = \frac{V_s^2 k}{R}$$

Power factor $\frac{V_r^2/R}{V_s \times I_r} = \frac{V_r^2/R}{V_s \times \frac{V_r}{R}}$

$$= \frac{V_s\sqrt{k}}{V_s}$$

$$\boxed{P.F = \sqrt{k}}$$

Multistage sequence control :-

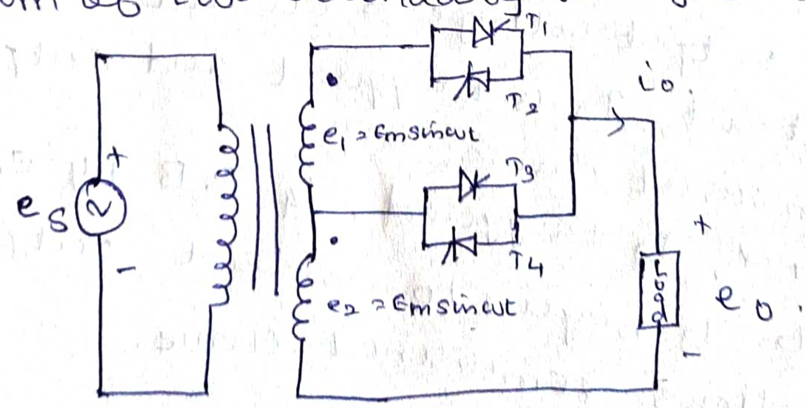
Two stage sequence control :-

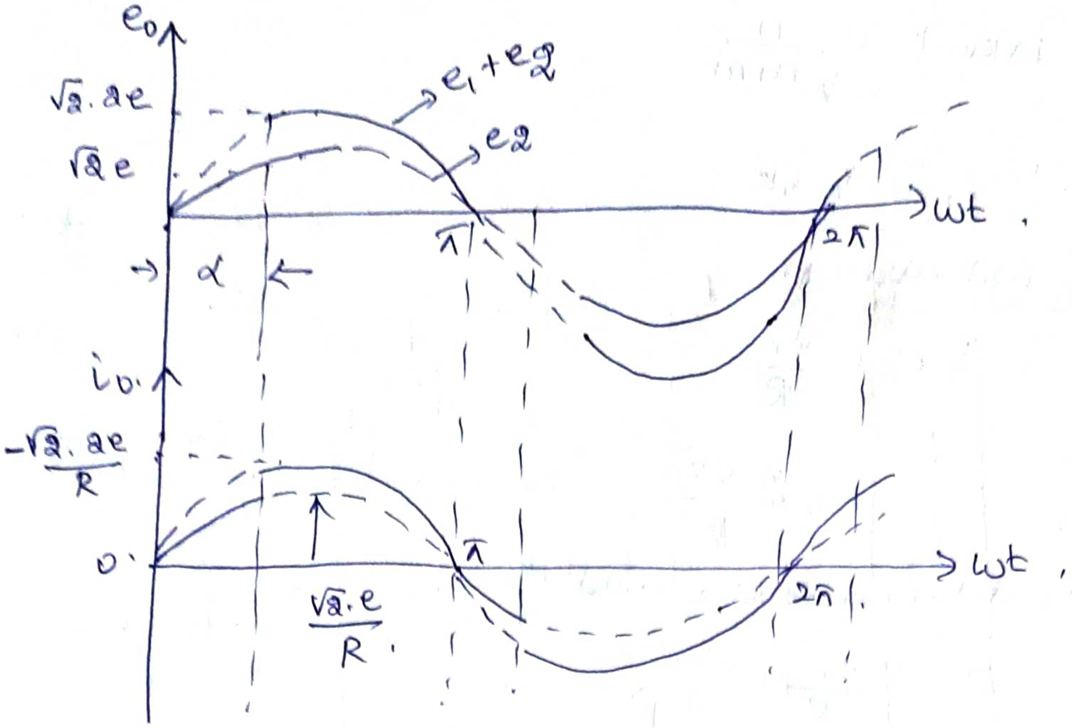
⇒ Two stage sequence control of a.c regulators employs two stages in parallel.

⇒ For source voltage $e_s = E_m \sin\omega t$,

$e_1 = e_2 = E_m \sin\omega t$.

sum of two secondary voltages is $2E_m \sin\omega t$.





⇒ For R load, load current waveform is identical with output voltage waveform.

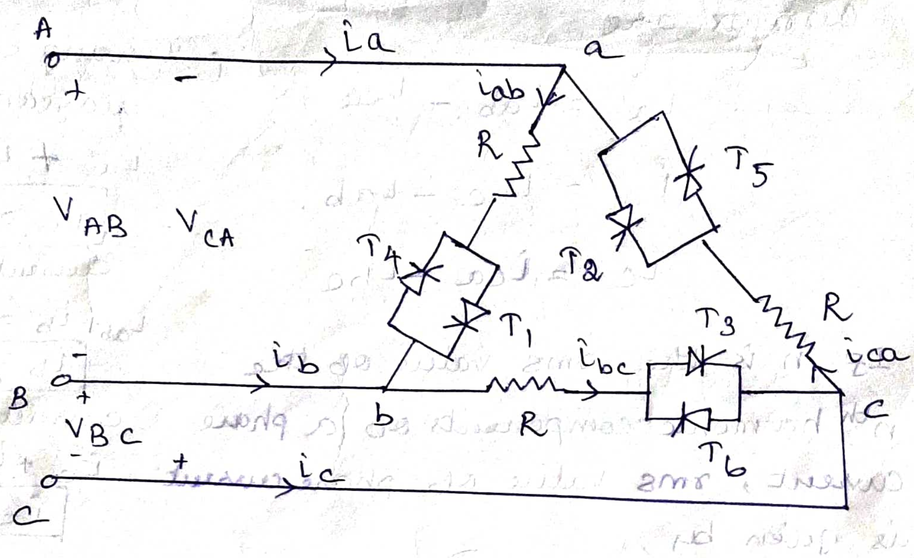
⇒ when both pairs T_1, T_2 & T_3, T_4 are in operation, firing angle for T_3, T_4 is zero,
T_1, T_2 varied from 180° to 0.
to obtain output voltage E to 2E.

⇒ T_3 triggered at $\omega t = 0$, follows $e_2 = E_m \sin \omega t$ curve. when SCR T_1 is triggered at $\omega t = \alpha$, voltage e_1 reverse biases T_3, it is turned-off. T_1 begins to conduct, output voltage jumps from e_2 to $(e_1 + e_2)$ and follows $2E_m \sin \omega t$ curve.

⇒ at $\omega t = \pi$, output voltage & current are zero.
at this instant, T_4 is triggered and output voltage follows $E_m \sin \omega t$ curve.

⇒ at $\omega t = \pi + \alpha$, T_2 triggered, T_4 R.B by $E_m \sin \alpha$, so, T_4 - off. T_2 begins to conduct, output voltage follows $2E_m \sin \omega t$ curve.

Three phase Bidirectional Delta connected controllers :



load may be connected in delta, The phase current in a normal 3ϕ is only $\frac{1}{\sqrt{3}}$ of the line current.

Instantaneous line-to-line voltages are

$$V_{AB} = \sqrt{2}\, V_s \cdot \sin \omega t$$

$$V_{BC} = \sqrt{2}\, V_s \cdot \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{CA} = \sqrt{2}\, V_s \cdot \sin\left(\omega t - \frac{4\pi}{3}\right).$$

For resistive loads, rms output phase voltage can be determined from,

$$V_0 = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} V_{ab}^2\, d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{\pi}\int_{\alpha}^{\pi} (\sqrt{2})^2 (V_s \sin \omega t)^2 \cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{\pi}\int_{\alpha}^{\pi} 2 V_s^2 \sin^2 \omega t \cdot d\omega t\right]^{1/2}$$

$$= \frac{\sqrt{2}\, V_s}{\sqrt{\pi}}\left[\int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} \cdot d\omega t\right]^{1/2}$$

$$V_{orms} = \frac{V_s}{\sqrt{\pi}}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi} = \frac{V_s}{\sqrt{\pi}}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}.$$

line currents can be determined from the phase currents are,

$$i_a = i_{ab} - i_{ca}$$
$$i_b = i_{bc} - i_{ab}$$
$$i_c = i_{ca} - i_{bc}$$

(∵ using kcl in circuit diagram, current at node a,

$i_a + i_{ca} = i_{ab}$.

$\boxed{i_a = i_{ab} - i_{ca}}$

current at node b,

$i_{ab} + i_b = i_{bc}$

$\boxed{i_b = i_{bc} - i_{ab}}$.

current at node c,

$i_c + i_{bc} = i_{ca}$.

$\boxed{i_c = i_{ca} - i_{bc}}$

If n is the rms value of the n^{th} harmonic components of a phase current, rms value of phase current is given by,

$$I_{ab} = \left[I_1^2 + I_3^2 + I_5^2 + I_7^2 + \ldots\ldots I_n^2\right]^{1/2}.$$

Due to delta connection, the triplen harmonic components of the phase currents would flow around the delta and would not appear in the line. Hence,

$$I_a = \sqrt{3}\left[I_1^2 + I_5^2 + I_7^2 + \ldots\ldots I_n^2\right]^{1/2}.$$

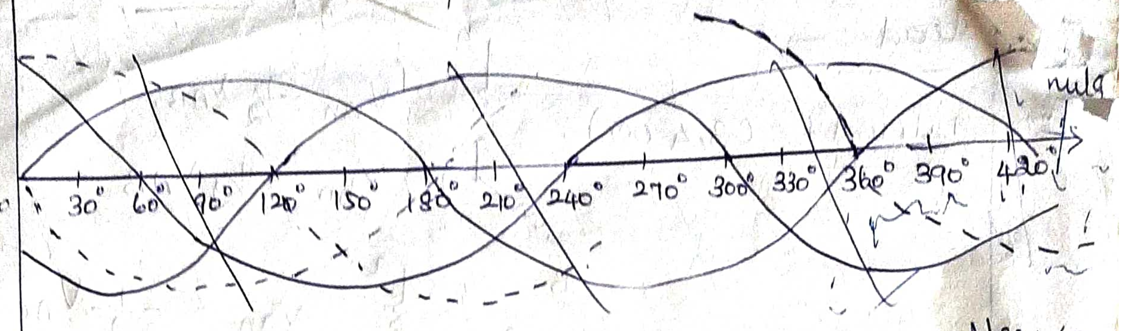
$$I_a < \sqrt{3}\, I_{ab}$$

⇒ SCR are rated to carry phase currents and withstand the line voltage.

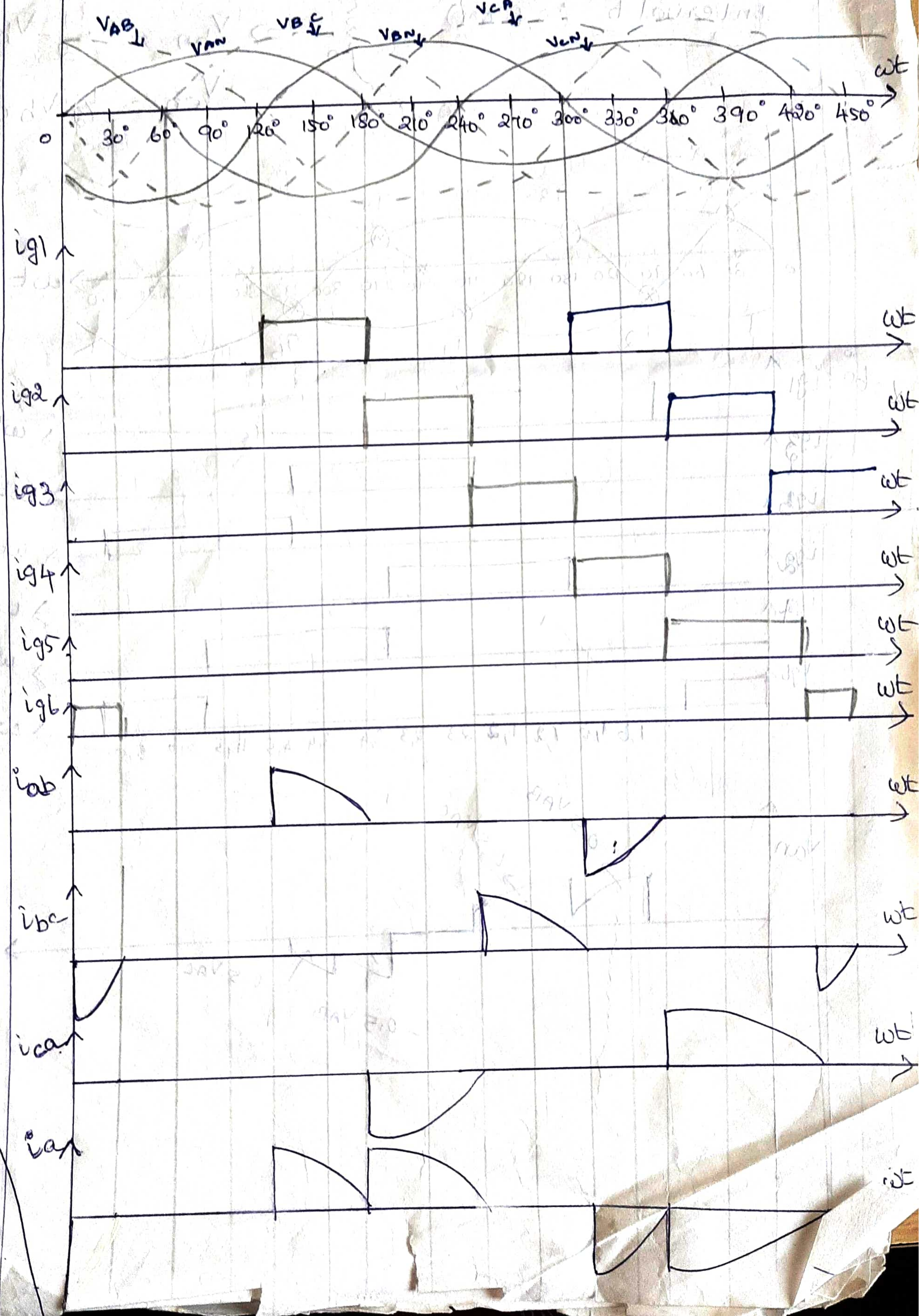
⇒ The power factor is slightly higher.

⇒ The voltage across an R-load is the corresponding line-to-line voltage when one SCR in that phase is on.

⇒ The firing angle α is measured from the zero crossing of the line to line voltage and the SCR turned on in the sequence they are numbered.



3φ Bidirectional delta connected controllers:-



α=120°

Draw ib, ic also

Final fall time (t_{f2}) :-

The time during which collector current falls from 20 to 10% of I_c, or the time during which collector emitter voltage rises from V_{CES} to $0.1V_{CE}$. The final fall time (t_{f2}) is the time during which collector current falls from 20 to 10% of I_c, or the time during which V_{CE} rises from $0.1V_{CE}$ to final value V_{CE}.

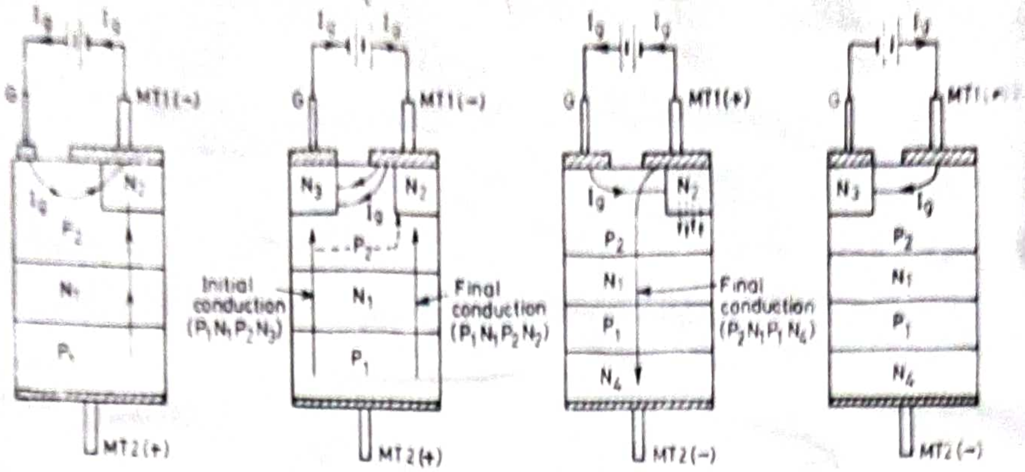
The Triac :

A triac is a bidirectional thyristor with three terminals. It conducts in both the directions. When in operation, a triac is equivalent to two SCRs connected in antiparallel. The three terminals are MT_1 (main terminal 1), MT_2, and the Gate (G).

Cross sectional view of a Triac :

The Gate G is near terminal MT_1. The G is connected to N_3 as well as P_2. Terminal MT_1 is connected to P_2 and N_2; terminal MT_2 is connected to P_1 and N_4.

With no signal to gate, the triac will block both half cycles of the ac applied voltage in case peak value of this voltage is less than the breakover voltage of V_{BO1} or V_{BO2} of the triac. Terminal MT_1 is taken as the point for measuring the voltage and current at the gate and MT_2 terminals.



turn on process of a triac:

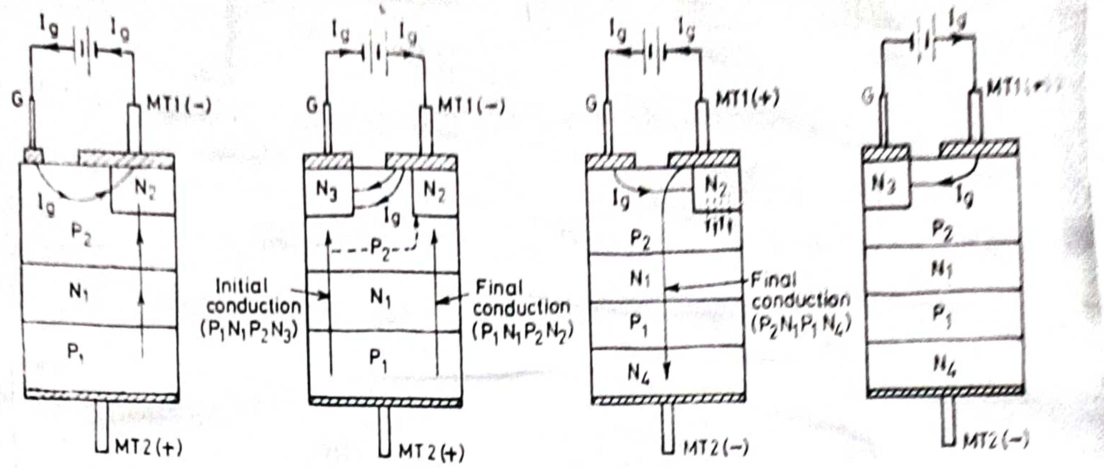
(i) MT_2 is positive and gate current is also positive:

When MT_2 is positive with respect to MT_1, Junction P_1N_1; P_2N_2 are forward biased but junction N_1P_2 is reverse biased. When gate terminal is positive with respect to MT_1, gate current flows mainly through P_2N_2 junction like an ordinary SCR.

When gate current has injected sufficient charge into P_2 layer, reverse biased junction N_1P_2 breaksdown as in a normal SCR. Triac starts conducting through $P_1N_1P_2N_2$ layers. triac operates in the first Quadrant.

(ii) MT_2 is positive but gate current is negative:

When gate terminal is negative with respect to MT_1, gate current flows through P_2N_3 Junction. triac starts conducting through $P_1N_1P_2N_3$ layers initially. with the conduction of $P_1N_1P_2N_3$, the voltage drop across this path falls but potential of layer between P_2N_3 rises towards the anode potential of MT_2. Left hand region being at higher potential than its right hand region. Right hand part of triac consisting of main structure $P_1N_1P_2N_2$ begins to conduct.



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(iii) MT_2 is negative but gate current is positive :

The gate current I_g forward biases P_2N_2 junction. Layer N_2 injects electrons into P_2 layer. Reverse biased junction N_1P_1 breaks down. The structure $P_2N_1P_1N_4$ is completely turned ON. The triac is turned on by remote gate N_2, the device is less sensitive in the third Quadrant with positive gate current.

(iv) Both MT_2 and gate current are negative :-

The gate current I_g flows from P_2 to N_3. Reverse biased junction N_1P_1 is broken and finally the structure $P_2N_1P_1N_4$ is turned on completely.

Triac with voltage and current ratings of 1200V and 300 A (rms) are available.

A triac operate in the rectifier mode than in the bidirectional mode, due to following reasons :

(a) For a given value of +ve gate current, a triac may turn ON with MT_2 (+ve) in first Quadrant but fail to turn ON with MT_2 (−ve).

(b) With constant negative gate current ; the triac may turn on with MT_2 (−ve) in third quadrant but may not turn on with MT_2 (+ve).

The rectifier mode can be overcome by increasing gate current.